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Please Note:

RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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NOTICING NUMERACY NOW (N^3): A COLLABORATIVE RESEARCH PROJECT TO DEVELOP PRESERVICE TEACHERS' ABILITIES TO PROFESSIONALLY NOTICE CHILDREN'S MATHEMATICAL THINKING

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The goal of the Noticing Numeracy Now (N^3) research project is to determine the extent to which an innovative learning experience focused on the professional noticing of children's early numeracy thinking develops preservice teachers' capacity to attend to, interpret, and respond appropriately to children's mathematical thinking. The N^3 project is being implemented at eight Kentucky public universities.

Historically, preservice elementary teachers (PSETs) demonstrate diverse and uneven conceptualizations of key ideas related to the effective teaching and learning of mathematics often coupled with negative or ambivalent attitudes towards the discipline. This paper provides information about the extent to which an innovative learning experience focused on the professional noticing of children's numeracy develops PSETs' capacity to attend to, interpret, and respond appropriately to the mathematical thinking of children. The research uses a module, ***Noticing Numeracy Now (N^3)***, developed by the researchers and based on professional literature in the areas of professional noticing (Jacobs, Lamb, & Phillip, 2010) and the Stages of Early Arithmetic Learning (SEAL) (Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe, Cobb, & Glasersfeld, 1988; Steffe, 1992).

This collaborative effort builds on the expertise and experiences of post-secondary professors from Colleges of Education and Arts and Sciences and on the involvement of eight universities with strong and successful teacher education programs. Each university committed to institutionalize the activities as part of their elementary teacher preparation programs. The module is currently being used to present a creative and potentially transformative approach to the preparation of future elementary teachers via classroom and field activities that explicitly promote the development of the component skills of professional noticing in the context of SEAL. The research questions being investigated are:

1) To what extent can teacher educators facilitate the development of PSETs' capacity to professionally notice (attend, interpret, and decide) children's mathematical thinking?
2) To what extent can PSETs develop understanding of children's conceptions of unit as displayed by counting types in the Stages of Early Arithmetic Learning?
3) To what extent does PSET professional noticing performance correlate with PSET mathematical knowledge for teaching and attitudes towards mathematics?

Review of Literature

Professional Noticing: Challenges and Opportunities

While the preponderance of literature on PSETs' beliefs and attitudes reports that PSETs viewed mathematics negatively (Ball, 1990) or with neutrality (Quinn, 1997), PSETs generally hold positive perceptions of children and children's potential to learn, often reporting this as the underlying reason for their choice of careers. Philipp et al. (2007) suggest that capitalizing on this positive perception of children can be a persuasive tool for developing stronger and more positive attitudes for mathematics among PSETs. PSETs' noticing the personal strategies that children employ to solve mathematical problems indicated a belief that mathematics can be made sense of (Ambrose, 2010) rather than perceiving mathematics simply as a system of rules and procedures that must be transferred to students (Ball, 1990; Foss & Kleinsasser, 1996). Over the past decade, the literature exploring the construct of noticing in mathematics education has grown. Sherin and van Es (2009) have contributed much to the field through their work with teachers using video clubs as a tool for analyzing their classrooms. Numerous professional development modules have incorporated the use of video to focus observers' attention on children's mathematical thinking (Seago, Mumme, & Branca, 2004; Schifter, Bastable, Russell, 2000; Carpenter, Fennema, Franke, Levi, & Empson, 1999). This work has primarily focused on professional development of in-service teachers. One exception to this is the Integrating Math and Pedagogy (IMAP) project which used video with pre-service teachers (Philipp et al., 2007).

Stages of Early Arithmetic Learning (SEAL)

In the late 1970's, Steffe and his colleagues enacted a series of teaching experiments with young children to determine different types of quantitative understanding and how such understanding may change over time (Steffe et al., 1983; Steffe, et al., 1988; Steffe, 1992; Steffe

& Thompson, 2000). Effort was expended to ensure that these episodes focused on the students' mathematics rather than the mathematics of the teacher (Steffe and Thompson, 2000). These series of student-centered teaching experiments resulted in the description of distinct counting types (Steffe et al., 1983) and ultimately a progression of arithmetic stages (i.e. emergent, perceptual, figurative, etc.) predicated on the child's understanding of unit at a particular time (Steffe et al, 1988; Wright, Martland, & Stafford, 2000).

Noticing Numeracy Now (N^3) Module Description

Given the aim to promote professional noticing of children's mathematics among PSETs, the N^3 module was developed for inclusion in elementary teacher preparation programs. The module consists of five class sessions and emphasizes brief video-recorded child/teacher interactions as a context for the PSETs to incrementally develop the components of professional noticing (attending, interpreting, and deciding) as described by Jacobs, Lamb, and Philipp (2010). The video vignettes are designed to scaffold PSETs' construction of the numeracy progression of counting types outlined by the Stages of Early Arithmetic Learning (SEAL) (Steffe et al, 1983; Steffe, Cobb & von Glasersfeld, 1988). Complementing the video vignettes are multiple interactive classroom activities, homework assignments, and a culminating self-analysis interview assignment. Table 1 outlines the foci, instructional activities, and objectives of the five-session module.

Professional Noticing Interview Assignment

The PSETs practice their professional noticing skills by conducting an early numeracy diagnostic interview at an elementary school with a child in grades kindergarten through second grade. The PSETs videotape the interview and may work with another PSET and interview one student one to two times during the semester. For the interview, PSETs are provided resources from which to choose appropriate tasks. Examples of such resources include the elementary mathematics diagnostic screening tools developed by the Boulder Valley Schools Department of Learning Services, Colorado (BVSD, 2010). The design of these documents includes a separate interview for each grade level. To prepare for the interview, the PSETs are encouraged to use the current grade level document in addition to preparing opportunities for the K-2 child to perform at their individual level. In other words, the 1st grade document may be too rigorous for a 1st grade child chosen, and the PSET may supplement the interview with the kindergarten level interview. Similarly, the PSET may find the need to scaffold to the next grade level or beyond with the child being interviewed if the child is showing mastery of skills at the assigned grade level. Such decisions incorporate the components of professional noticing into this assignment.

Table 1.

Noticing Numeracy Now (N^3) Module Sessions Overview

Session	Professional Noticing Focus	Children's Mathematical Thinking Focus	Interactive Classroom Instructional Strategy	Objectives(s)
1	**Interview Process and Benefits**	Children's mathematical strategies for early numeracy concepts	**Group development** of a progression of children's mathematics' strategies	A. Establish a purpose for student interviews and familiarize preservice elementary teachers (PSETs) with interview process. B. Explore, through video clips, differing strategies that children may apply to particular tasks.
2	**Attending**	Concept of unit and counting types	**World Café:** focus on mathematics, children's thinking, teacher practices	A. Introduce the concept of attending to the mathematical thinking and actions of the individual child with respect to counting type and understanding of unit.
3	**Attending and Interpreting**	Stages of Early Arithmetic Learning: 0: Emergent, 1: Perceptual, 2: Figurative	**Think, Pair, Share:** Focus on hallmarks of SEAL 0, 1, 2	A. Reinforce the concept of attending to the mathematical thinking and actions of the individual child. B. Introduce the concept of interpreting the mathematical thinking and actions of the individual child. C. Gain familiarity with perceptual and figurative conceptualizations of unit and counting types.
4		Stages of Early Arithmetic Learning: 3: Initial, 4: Intermediate, 5: Facile Number Sequences	**Guess My Stage:** focus on hallmarks of SEAL 3, 4, 5	A. Reinforce the concepts of attending to and interpreting the mathematical thinking and actions of the individual child. B. Gain familiarity with children's construction of the initial number sequence, intermediate number sequence, and facile number sequence.
5	**Attending, Interpreting, and Deciding**	Examining the interviewer's decisions	**Role Playing:** SEAL & Professional Noticing and **Chorale Montage:** what have we learned?	A. Reinforce the concepts of attending to and interpreting the mathematical thinking and actions of the individual child. B. Introduce the process of interpretation-based decision-making with respect to different counting types and understanding of unit.

The pilot sites of the N^3 collaborative project have incorporated several variations of assessment of the PSETs' Diagnostic Interview Analysis. A sample assessment rubric with the targeted components of the assignment and criteria is provided below (see Table 2).

Through the interviews, the PSETs experience the rich opportunity for deep understanding of how a child thinks in terms of numeracy and have the task of determining the Stage of Early Arithmetical Learning (SEAL) at which the child is performing. In addition to the specific goal for the PSETs to develop professional noticing in the context of SEAL, some broader aims are for the PSETs to gain confidence, develop a sense of the value and importance of professional noticing, and generalize this to their broader teaching contexts.

Table 2.

Sample Assessment Rubric

Professional Noticing Component	Performance Criteria
Attending Component: *Five-minute transcript and between-the-lines analysis*	The transcript is clear and documents at least five minutes of the interview.
	The analysis of PSET questioning/discourse is evident.
	The analysis of student thinking is evident.
Interpreting Component – Part 1: *What PSET learned about self as a teacher*	PSET suggests specific ways to strengthen own questioning based on self-analysis of interview and supported with readings.
	PSET recognizes personal strengths in a balanced way.
Interpreting Component – Part 2: *What PSET learned about child's mathematical thinking*	PSET identifies child's conceptual and procedural understanding of the assessed concept.
	PSET interprets child's actions and responses in the context of SEAL.
	PSET supports child's understanding, or lack of understanding, of the assessed concept with evidence from interview and from readings.
Deciding Component: *Next steps*	PSET chooses appropriate next instructional step(s).
	PSET justifies next instructional step(s) with evidence from student's work and readings.

Research Questions and Methods

Returning to the research questions stated in the introduction, we designed a particular assemblage of methods to address each specific inquiry.

Research Methods – Question 1

Data for this portion of the research program will consist of pre- and post-module PSET responses to video-recorded and written exchanges between a teacher and an elementary student. These exchanges depict (or describe) a teacher, using particular tools, to pose one or more arithmetic tasks to a child. The child will then enact strategies indicative of a particular counting type and arithmetic stage. Having viewed or read this exchange, PSETs will be prompted to:

- describe important mathematical activities and features of the exchange *(attending);*
- describe how these activities and features are indicative of a particular conceptual understanding or counting type *(interpreting);* and
- describe appropriate next instructional steps for the student in question *(deciding).*

With respect to diversity at the participating institutions (i.e. rural, suburban, urban), a stratified random sample of these pre/post data will be scored according to an established noticing rubric (Jacobs et al., 2010) which provides a numerical value for each PSET response in the component skills of attending, interpreting, and deciding. Additionally, this sample of PSET responses will be coded and inductively analyzed with the aim of determining further emergent themes (Glaser & Strauss, 1967) with respect to the three component skills of professional noticing.

Research Methods – Question 2

As with measurements aimed at professional noticing, data from this portion of the research program also consists of PSETs' pre- and post-responses to video-recorded and written exchanges between a teacher and an elementary student. In this instance, though, primary attention is given to relationship between PSETs' responses in the interpreting and deciding domains. The focus here is: a) to what extent is the PSETs' interpretation consistent with the child' numeracy stage, and b) to what extent is the PSET instructional decision consistent with advancing the student along the SEAL progression (Steffe et al., 1988; Wright et al., 2000). A stratified random sample of PSET responses from each module implementation will be collected and all module implementers will assign a score of either 0 (remarks inconsistent with student's stage), 1 (remarks moderately consistent with student's stage) or 2 (remarks highly consistent with student's stage). Additionally, the same sample of PSET responses will be scored by all module implementers using the same scale.

Research Methods – Question 3

PSETs will be assessed using the Learning Mathematics for Teaching (LMT) assessment (Hill & Ball, 2004) both before the module and at the conclusion of the semester. This assessment will be administered online and responses will be routed to the module implementers. The LMT is primarily designed to measure changes in different types of mathematical knowledge including content knowledge and Mathematical Knowledge for Teaching and has been the subject of rigorous development and testing. The principal investigators, co-investigator, and project evaluator will be trained in the administration of the LMT prior to measured module implementation. Additionally, PSET attitudes and beliefs regarding mathematics will be measured using the Attitudes Towards Mathematics Inventory (ATMI) (Tapia, 1996; Tapia & Marsh, 2004; Schackow, 2005) which was selected based on an extremely high level of reliability (α=.97). This 40-item inventory will be administered either online or in-person and results will be routed to or collected by module implementers.

Timeline and Conclusion

This project contains three phases: 1) design/pilot (fall 2009-fall 2010) 2) primary-implementation and (spring 2011-fall 2012) and 3) summative (spring 2013). Of particular interest is the primary implementation phase where the project team will collect and analyze data from implementations and leverage these data for module refinement. Additionally, outcome dissemination efforts will begin during this phase. Given the importance of developing highly-skilled teachers of mathematics, our intention with this research effort is to illuminate the potential of professional noticing as it pertains to developing PSETs' capacity to attend to, interpret, and respond appropriately to the mathematical thinking of children. We believe that this capacity directly correlates to children's robust early mathematical development.

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EXPLORING PROSPECTIVE ELEMENTARY TEACHERS' ABILITIES TO SOLVE NON-ROUTINE PROBLEMS: CONTENT, COGNITIVE LEVEL, AND HABITS OF MIND

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This study explored fifty-five prospective elementary teachers' ability to solve non-routine mathematics problems with respect to content area, cognitive levels, and their problem-solving habits of mind. Problems were categorized by level of cognitive ability (Smith & Stein, 1998) and content area addressed in the problem. Reflective journals written by the prospective teachers were coded for themes indicating the habits of mind they employed. Results indicate they had the most difficulty in solving the "procedures with connections" type algebra and geometry non-routine problems, but cultivated many of the habits of mind necessary for problem solving.

The National Mathematics Panel (2008) states that all mathematics classrooms need to emphasize conceptual understanding, computational fluency, and problem solving across grade levels. In order to create classrooms with a problem-solving culture, teachers need to select cognitively demanding problems that require students to think and make sense of the mathematics, use multiple strategies to solve, explain and justify their strategies, and engage in discussions about how those strategies are mathematically related (National Council of Teachers of Mathematics [NCTM], 2000; National Research Council, 2001).

Also, empirical evidence shows that teachers' subject matter knowledge impacts student achievement (Goldhaber & Brewer, 2000). Prospective teachers need a solid foundation of the mathematics they will teach and need to understand how to solve problems using this content. By engaging in problem-solving experiences, these future teachers can sharpen their understanding of mathematics and learn how to connect mathematical concepts and ideas (Grossman & Schoenfeld, 2005, p. 216). The prospective teachers also need to be aware of their own problem-solving habits of mind and the role they play in helping them become effective problem solvers. As future mathematics teachers, they "must teach the habits of mind, model the habits of mind, train students to use the habits of mind, and make students aware when they are engaged in the habits of mind" (Wilburne, 2006, p. 15).

Theoretical Framework

There has been a plethora of research on mathematical habits of mind. Cuoco, Goldenberg, and Mark (1996) describe *habits of mind* as "mental habits that allow students to develop a repertoire of general heuristics and approaches that can be applied in many different situations" (p. 378). They posit that teaching students habits of mind help them learn to think like mathematicians. Levasseur and Cuoco (2003) and Goldenberg, Sheingold, and Feurzeig (2003) proposed mathematical habits of mind for secondary students and young children. Even the new Standards for Mathematical Practice in *Common Core State Standards in Mathematics* (CCSSI, 2010, p. 6-8) identify a list of habits of mind necessary to reason and think mathematically.

Costa & Kallick (2000) describe 16 habits of mind that are common across solving problems for which the solution is not obvious. They include: (1) persisting; (2) managing impulsivity; (3) listening to others; (4) thinking flexibly; (5) thinking about your thinking; (6) striving for accuracy and precision; (7) questioning and posing problems; (8) applying past knowledge to new situations; (9) thinking and communicating with clarity and precision; (10) gathering data through all senses; (11) creating imagining, and innovating; (12) responding with wonderment and awe; (13) taking responsible risks; (14) finding humor; (15) thinking interdependently; and (16) learning continuously. Schoenfeld's (1985) work described problem-solving habits of mind of expert problem solvers versus novice problem solvers. Thinking metacognitively, having a positive attitude, refraining from impulsivity, asking questions, being flexible, and seeking clarity are habits of mind that helped expert problem solvers think through a problem and successfully solve it. The *Principles and Standards for School Mathematics* (NCTM, 2000) state, "A problem-solving disposition includes the confidence and willingness to take on new and difficult tasks. Successful problem solvers are resourceful, seek out information to help solve problems and make effective use of what they know. Their knowledge of strategies gives them options. If the first approach to a problem fails, they can consider a second or a third. If those approaches fail, they know how to reconsider the problem, break it down, and look at it from different perspectives—all of which can help them understand the problem better or make progress toward its solution. Part of being a good problem solver is being a good planner, but good problem solvers do not adhere blindly to plans." (NCTM, 2000, p. 334).

The Conference Board for Mathematical Sciences (CBMS, 2001) recommends for prospective elementary teachers' mathematics content courses to engage them in problems designed to promote their problem-solving ability and help them identify the skills and habits of mind necessary to think through a problem. Also, it is important to make prospective teachers aware of the different levels of cognitive demand that are required to solve problems. Smith and

Stein (1998) describe four levels of cognitive demand for mathematical tasks: memorization; procedures without connections; procedures with connections; and doing mathematics. They posit that different tasks provoke different levels and kinds of student thinking. Thus, if we want students to develop the capacity to think, reason, and problem solve then we need to engage them in high-level, cognitively complex tasks (Stein & Lane, 1996).

Our research aimed to examine how successful prospective elementary teachers were with solving non-routine problems identified with different content areas at differing levels of cognitive demand (Stein & Smith, 1998). We also wanted to identify the problem-solving habits of mind prevalent among prospective elementary teachers as reported in their reflective journals and map them to those of Costa & Kallick (2000), Schoenfeld (1985), and the NCTM Standards (2000). If we know what problem-solving habits of mind prospective teachers apply, we can make them cognizant of these dispositions as well as cultivate the habits of mind they are not aware of. This in turn should help them promote effective problem-solving behaviors with their future students. We explored two research questions: (1) *What types (content and cognitive ability) of non-routine problems are elementary prospective teachers successful in solving? And (2) What habits of mind do prospective elementary teachers identify as beneficial to successfully solve non-routine problems?*

Methodology

Data were collected from 55 prospective elementary teachers enrolled in mathematics content courses at two different mid-atlantic universities during the Spring 2010 semester. Prospective teachers were given sets of non-routine problems that addressed a variety of topics from the elementary curriculum and posed challenging mathematical questions. It was decided to use problems from the Math Olympiad for Elementary and Middle School (MOEMS, www.moems.org). The Division E MOEMS problems are mathematical contests geared for elementary school students in grades 3-6. The data consisted of six sets of MOEMS contests[1], each set containing five problems. The sets were distributed weekly to the prospective elementary teachers for homework, and they were usually given one week to return the problems with full explanations of their work. In addition, end-of-the-semester reflections of these weekly

[1] Permission to use these Contests was provided by Richard Kalman, Executive Director of MOEMS. The contests used were: November 18, 2008; December 16, 2008; January 12, 2009; November 17, 2009; December 15, 2009; January 12, 2010.

problems were collected from one of the sections. Students were asked to reflect on: (a) their overall thoughts about the problems; (b) what learning they experienced through the problems; (c) what problems they enjoyed doing; (d) what strategy they used most often; and (e) how the experience of doing the non-routine problems can benefit them as future elementary teachers.

After each set of the problems were collected, the prospective teachers' answers were identified as being: (1) fully correct, (2) strategy correct but with incorrect answer, or (3) incorrect. For the purposes of this study, responses scored in the first two categories were counted as successful. The thirty problems (6 sets of 5 questions) were coded and analyzed in terms of their content and task level (Smith & Stein, 1998).

The content of the problems were collapsed into the following broadly defined content with the number in parenthesis: Number Theory (10), Algebra (7), Geometry (7), Probability & Statistics (3), Number & Operation (2), and Logic (1). Since the last three types of content areas contained too few questions to draw any conclusions, this study focused on Number Theory, Algebra, and Geometry type problems. The Number Theory problems focused on properties of numbers, such as place value, using divisibility rules, or being prime. For example, one question was: "111,111 is the product of 5 different prime numbers. What is the sum of those 5 prime numbers?" (MOEMS, January 12, 2010-#3E). The Algebra category included problems that could be solved using algebraic symbols and equations, and also included problems attending to prerequisite skills necessary to solve algebraic equations, such as ratios and number patterns. The first type of algebra problem could be solved using equations, but given that the target audience for the problems are 3rd through 6th grade students, also could be solved in other ways. For example: "One hat and two shirts cost $21. Two hats and one shirt cost $18. Megan has exactly enough money to buy one hat and one shirt. How much money does Megan have?" (MOEMS, January 12, 2010-#3B). A second type of Algebra problem relied on using ratios or proportions, such as: "It takes 3 painters 4 hours to paint 1 classroom. How many hours does it take 1 painter to paint 2 classrooms of the same size as the first one? Assume all painters work at the same rate." (MOEMS, January 12, 2010-#3C). Geometry problems fell into two subcategories, problems that dealt with measurement, such as surface area, and those that dealt with attributes of shapes. An example of the former was: "Three identical cubical boxes form a stack (diagram provided). It takes 350 sq cm of wrapping paper to completely wrap the whole stack with no overlap. Suppose each cube is wrapped separately and completely instead. What is the least amount of additional paper that is needed, in sq cm?" (MOEMS, December 16, 2008-#2E). An example of the latter was: "A rectangular box has a top that is 15 cm by 20 cm and a height of 4 cm. An ant begins at one corner of the box and walks along the edges. It touches all eight corners. What is the shortest distance, in cm., that the ant may travel?" (MOEMS, November 17, 2009-#1E).

The problems were also categorized according to the level of cognitive demand required for the task, namely: *memorization* (0), *procedures without connections* (18), *procedures with connections* (11), and *doing mathematics* (1) (Stein & Smith, 1998). It was no surprise that there were no *memorization* type problems considering the purpose of MOEMS is to challenge learners. The *procedures without connections* problems spanned all six of the content areas. An example includes the hat/shirt problem noted above, because it can be solved in a variety of ways but it does not necessarily connect different mathematics areas and can be solved in procedural ways. The 11 problems that fit the *procedures with connections* spanned five of the six content areas, not including the Logic category. Examples of these problems include the Number Theory example and both Geometry examples noted above. Although each of these problems have procedures that might guide some aspect of the solution (e.g., finding surface area), additional critical thinking is required in order to determine how the solution might be achieved. The one *doing mathematics* problem identified was: "A digital timer counts down from 5 minutes (5:00) to 0:00 one second at a time. For how many seconds does at least one of the three digits show a 2?" (MOEMS, November 17, 2009-#1D). This problem integrates patterns, measurement with respect to time, and number theory and requires no particular procedure to follow to solve it.

The reflective journals were coded using the habits of mind (Costa & Kallick, 2000) and expert problem-solving (Schoenfeld, 1985) frameworks. After analyzing these data, it was further noted how they compared to the problem-solving process standard (NCTM, 2000).

Findings

The data revealed the types of non-routine problems prospective teachers struggled with and the habits of mind they found that benefitted their ability to solve these problems. Table 1 incorporates the three content areas, the cognitive level of the problems, and the percentage of students who correctly answered the problem of those attempting them (n=48 to n=55). The entries italicized and bolded in the table identify the percentage of prospective teachers answering 80% or less correct.

Table 1.

Problem classifications with percent of prospective teachers' success

	Cognitive level of problem		
Content Area of Problem	Procedures without connections	Procedures with connections	Doing Mathematics
Number Theory (10)	100, 98, 96, 92, 90, 83, 82, ***76***	82, ***75***	
Algebra (7)	96, 88, ***79, 73***	***65, 61***	***65***
Geometry (7)	87, 82, ***77***	92, ***80, 77, 58***	

Content

In terms of the content, algebra and geometry caused the most difficulty for the prospective teachers. Only 2/7 of the algebra and 3/7 of the geometry problems were solved correctly by more than 80% of the prospective teachers. The algebra and geometry problems that students had difficulty with included solving multi-step equations, patterns, and finding the nth terms.

Cognitive Ability

With respect to the cognitive ability, the prospective teachers were successful in solving 11/15 of the procedures without connections problems, but only 2/8 of the procedures with connections problems. The procedures with connections problems required the prospective teachers to apply their knowledge of the content along with some critical thinking. To elaborate, one procedures with connections problem required students to determine the fewest number of 1' X 1', 2' X 2', and 3' X 3' tiles that could be used to tile a 5' X 5' floor (MOEMS, January 12, 2010) was solved successfully by all but four students. However, when students were asked to find the shortest distance an ant travels along a rectangular prism (MOEMS, November 17, 2009), which is a procedures with connections problem, more than 40% of the students struggled. They were able to draw the diagram but could not make the necessary connections needed to find the relationship between the sides and the surface area. Also, the only problem classified as doing mathematics was successfully solved by 65% of the prospective teachers. The higher cognitive level problems posed greater challenges and difficulty for these future teachers.

Habits of Mind

Table 2 displays the mapping of the habits of mind expressed by the prospective teachers in their reflections about their problem-solving experience to those identified by Costa and Kallick (2000), Schoenfeld (1995), and the NCTM (2000) problem-solving dispositions.

Table 2.

Mapping of habits of mind

	Habits of Mind		
Prospective Teachers' Description	Costa & Kallick	Schoenfeld	PSSM
"Don't give up"	Be persistent		
"Take time, don't rush through it"	Managing impulsivity	X	
"Turn the problem into something manageable"	Think flexibly	X	X
"Think about your solution and strategy"	Metacognition	X	X
"Need to have a solid content base"	Apply past mathematics knowledge to new situations		X
"Write out your strategies/solutions"	Thinking & communicating with clarity & precision	X	
"Think outside the box"	Creating, imagining, and innovating		
"Be positive, look forward to it"	Responding with wonderment and awe	X	X
"Okay to try one strategy and if it doesn't work try another"	Taking responsible risks		
"Be willing to ask for help"	Thinking interdependently	X	X
"Don't feel incompetent, there will always be a way to solve it"	Learning continuously		
"Be sure you understand the problem"		X	
"Consider different strategies"		X	X

The prospective teachers' reflective journals identified various habits of mind helpful to their problem-solving process. Each of the habits of mind they identified can be mapped to either ones described by Costa and Kallick (2000), Schoenfeld (1985), or the *Principles and Standards for School Mathematics* (NCTM, 2000). These habits of mind align with those espoused in the literature on problem solving and support the importance of these dispositions to be successful problem solvers. For example, students recognized the need to think about the problem and the possible strategies, and to try different strategies if the others didn't work. These habits of mind comport with those of Schoenfeld's research (1985) on expert problem solvers and their approaches to problem solving. Other students noted the importance of being confident and having a positive attitude about problem solving as the *Principles and Standards for School Mathematics* (NCTM, 2000) identify as important dispositions for problem solvers. This research supports the specific habits of mind employed when individuals engage in non-routine problem and contributes to the conversation of the need to emphasize these habits of mind when teaching problem solving. The study reveals how the use of weekly non-routine problems made prospective teachers cognizant of the various habits of mind they engaged in to solve such problems.

Conclusion

Overall, the prospective elementary teachers noted the challenges they faced and the joys they felt when they were successful in solving challenging problems. They learned new approaches to solving problems and made mathematical connections between their prior knowledge and the content learned in their elementary mathematics content course, and began to see how repeated practice with the non-routine problems enhanced their confidence with solving mathematical problems. As future teachers, it is important they understand the need to select worthwhile mathematical tasks that require all levels of cognitive demand and problems that will enhance their students' mathematical knowledge and challenge them. They must also understand the need to emphasize a culture of learning where problem-solving habits of mind are modeled and discussed. The role of the mathematics teacher educators is to support these understandings; one way to accomplish this is through incorporating non-routine problems in prospective teacher preparation, especially ones that focus on procedures with connections problems in the areas of algebra and geometry.

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EXPLORATORY ANALYSIS OF KOREAN ELEMENTARY PRESERVICE TEACHERS' PERSONAL EFFICACY AND OUTCOME EXPECTANCY IN MATHEMATICS TEACHING

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This study investigated Korean elementary preservice teachers' efficacy beliefs in mathematics teaching. Data were collected by means of the Mathematics Teaching Efficacy Beliefs Instrument for 106 elementary preservice teachers in Korea. Analysis of data revealed that preservice teachers at the end point of the program had significantly lower personal efficacy and outcome expectancy in mathematics teaching than those of preservice teachers at the beginning of the program. Possible reasons for these results are heavy course work in mathematics and socio-cultural influences such as strong parental support and prevailing private education.

Teacher efficacy has been considered as an important theoretical construct in teacher education over the past 25 years. Teacher efficacy was adapted from social cognitive theory of Bandura (1977, 1986, 1997), and it was defined as "a teacher's judgment of his or her capabilities to bring desired outcomes of student engagement and learning, even among those students who may be difficult or unmotivated" (Bandura, 1977; Cited from Tschannen-Moran & Hoy, 2001, p. 783).

Teacher efficacy is an influential factor in a teacher's instructional effectiveness (Coladarci, 1992; Ghaith & Yaghi, 1997; Soodak & Podell, 1993), and in students' academic achievement (Moore & Esselman, 1992; Ross, 1992) and motivational growth (Midgley, Feldlaufer, & Eccles, 1989). Because teacher efficacy has been defined as both context specific and subject-matter specific (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998), mathematics teaching efficacy is assumed to predict preservice teachers' future ability of teaching mathematics. Enochs, Smith, and Huinker (2000) developed the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) for elementary preservice teachers. Using the MTEBI, research revealed that mathematics methods courses and field experiences are factors in mathematics teaching efficacy (Utley, Bryant, & Moseley, 2005; Wenner, 2001), mathematics instructional strategies are associated with mathematics teaching efficacy (Swars, 2005), and mathematics teaching efficacy and mathematics anxiety are negatively related (Gresham, 2008; Swars, Smith, Smith, & Hart, 2009).

Teacher efficacy has been studied mostly in the US and other western cultures; a few studies have been conducted on teacher efficacy in non-Western cultures. For example, Alkhateeb (2004) translated the MTEBI into Arabic to verify its accuracy in Jordan. Cakiroglu (2008) found that Turkish preservice teachers tend to have stronger outcome expectancy than the US preservice teachers. Ryang (2007) reported that the MTEBI's two-factor structure may not be an appropriate fit for Korean preservice teachers.

Understanding preservice teachers' efficacy beliefs is an important factor in knowing how or whether new teachers will succeed in their practice. The purpose of this study is to investigate mathematics teaching efficacy beliefs of Korean elementary preservice teachers. The research questions guiding this study are:

(1) Is the MTEBI appropriate for use with Korean elementary preservice teachers?
(2) How do the mathematics teaching efficacy beliefs of Korean elementary preservice teachers differ from the beginning to the end of their teacher education program?

Theoretical Framework

Bandura (1977, 1986) constructed self-efficacy in his social cognitive theory, and he asserted that an individual's future behavior can be more accurately predicted through one's self-efficacy than through past accomplishments. According to his theory, an individual's behavior is influenced by both self-efficacy and outcome expectancy. Self-efficacy is an individual's beliefs that influence his or her capability to cope with change in situated experiences, while outcome expectancy is a generalized expectation that influences an individual's action-outcome contingencies based on perceived life experiences. Both self- efficacy and outcome expectancy differentiates the situation and the context of that situation (Bandura, 1986, 1997).

The MTEBI used in this study—which was developed based on Bandura's (1986) theory—consist of the two scales: the Personal Mathematics Teaching Efficacy (PMTE) and the Mathematics Teaching Outcome Expectancy (MTOE). The PMTE scale identifies beliefs in a teacher's capability to teach mathematics effectively; the MTOE scale identifies a teacher's beliefs that effective teaching will have a positive influence on students' mathematics achievement. The PMTE and MTOE scales correspond respectively to Bandura's self-efficacy and outcome expectancy.

Method

Settings

The Korean elementary teacher education program in this study is a 4-year program exclusively run by a national university of education. This university houses many departments relating to the different subjects taught in elementary schools. The first year of the elementary teacher education program covers general education and prepares them for the further study in a specific subject area. Although elementary preservice teachers are supposed to teach all subjects in a school, they enroll in a specific track in their sophomore year to focus on one subject more than the others.

To complete the program, elementary preservice teachers need to earn 140 credit hours for graduation. At least 120 credit hours must come from liberal arts, pedagogy, contents, methods, electives, and field experience. They also take content and method courses in various subjects. A student in the mathematics track takes an additional 14 credit hours of courses in mathematics such as Calculus, Set theory, Modern Algebra, Analysis, Geometry, Topology, and Statistics. Students also take six credit hours of methods courses such as History of Mathematics Education, Mathematics Teaching Materials, and the Psychology of Learning Mathematics. In addition to coursework, an elementary preservice teacher has clinical experiences at various points in the program. A typical format consists of a 2-week observation in the sophomore year, a 2-week participation in the junior year, and a 4-5 week professional practice in the senior year.

Participants

The participants are of 106 Korean elementary preservice teachers at a teacher education (mathematics further track) program. Because freshmen have not yet chosen a subject area track, they are assumed to be conceptually different from the sophomores, juniors and seniors. Thus, the sample was composed of 35 sophomores, 30 juniors, and 41 seniors. The average age of participants was of 23.12 years (SD = 3.41). One third (33%) of participants were male; others (67%) were females.

Instrumentation

The instrument used in this study was the MTEBI, which consists of 21 items with each item having five choices: Strongly Disagree (SD), Disagree (D), Uncertain (UN), Agree (A), and Strongly Agree (SA). Responses were scored using a Likert scale from 1 for SD though 5 for SA. The instrument was translated into Korean by the author and two qualified bilingual graduate students. Some items were modified in order to fit the Korean language and culture. Each item was coded by the item number with P (indicating personal efficacy) or O (indicating outcome expectancy). For example, P2 indicates that Item 2 belongs to the PMTE scale; O1 indicates that Item 1 is of MTOE.

Data Analysis

After exploring the factor structure using principal component analysis (PCA), one-factor analysis of variance (ANOVA) model was used to analyze the data. With respect to the normality assumption of an ANOVA model, although there is no unified agreement on the minimal size of a group, 25 individuals per group is considered acceptable (Lomax, 2001). Because the three groups—those are sophomores, juniors, and seniors—in this study had more than 30 subjects, the group sizes were considered appropriate.

Results

Factor Analysis

PCA was used to explore the two-factor model on the data. The KMO index was .84, and Bartlett's sphericity test significant ($\chi^2 = 819.78$, $df = 210$, $p < 0.001$). PCA with promax rotation on the whole 21 items extracted six eigenvalues, 6.61, 2.11, 1.44, 1.21, 1.13, 1.03, greater than 1. The first two eigenvalues were distinctively higher than the others which gradually decreased. By Kaiser and Cattell's suggestions (Hill & Lewicki, 2007), it was considered that the instrument has a two-factor structure. However, the pattern matrix of PCA showed that items P2 and O14 had loadings to the crossover components. These two items impacted the validity of the instrument. After deleting the two items, PCA with promax rotation was performed again with 19 items. All P-initial items had loadings to the Component 1 and all O-initial items had loadings to the Component 2. Therefore, the Component 1 turned out to be the PMTE and Component 2 the MTOE (see Table 1). The PMTE scale had the extraction sum 6.08 and explains 32% of variance; the MTOE scale had the extraction sum 2.00 and explains 10.51% of variance. The alpha reliability indices of the two scales were .86 and .66, respectively.

ANOVA

A comparison of the mean scores of the three preservice teacher groups by Level (sophomore, junior, senior) was performed by one-way ANOVA on each scale where Level was a factor and PMTE or MTOE were the dependent variables. Levene's test of equality of error variance were respectively $F(2, 103) = 0.54$, $p = .58$ on PMTE and $F(2, 103) = 0.12$, $p = .88$ on MTOE indicating that PMTE and MTOE score distributions are not different across the groups. The ANOVA results (Table 2) indicated that Level has a significant effect on PMTE ($p = 0.036$), and MTOE (p = 0.018). The Bonferroni post hoc test revealed that there is a significant difference between sophomores and seniors on both the PMTE scores (p = .042), and the MTOE scores (p = .016).

To investigate the responses of preservice teachers to each item, multivariate ANOVA was conducted in each scale. The results indicated that the Level had no significant effect in the PMTE items (Wilks λ = .70, F = 1.50, p = .07, η^2= .16, $1 - \beta$ = .95), nor in the MTOE items (Wilks λ = .84, F = 1.27, p = .22, η^2= .08, $1 - \beta$ = .75). However, follow-up univariate ANOVA revealed that Items 1, 4, and 11 were statistically significant.

Table 1.
Principal Component Matrix

	Component	
Item	1	2
P15	.748	
P8	.741	
P19	.729	
P3	.722	
P21	.646	
P16	.606	
P18	.591	
P5	.565	
P6	.524	
P17	.492	
P11	.480	
P20	.470	
O9		.745
O10		.661
O13		.625
O12		.616
O7		.599
O4		.580
O1		.330

Table 2.

Analysis of Variance for the PMTE and MTOE

Source	*SS*	*df*	*MS*	*F*	*p*	η^2	1 - β
PMTE							
Level	236.08	2	118.04	3.43	.036	.062	.632
Error	3547.06	103	34.44				
Total	205407.00	106					
MTOE							
Level	76.56	2	38.28	4.20	.018	.075	.726
Error	938.88	103	9.12				
Total	68370.00	106					

Note. *SS* = Sum of square, *df* = Degree of freedom, *MS* = Mean square, β = Type II error

Discussion

Factor Structure of the Instrument Responses

In this study, the result of principal component analysis indicated that two items violate the two-factor structure of the instrument. These two items are Item 2 (*I will continually find better ways to teach mathematics*), and Item 14 (*If parents comments that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher*). After deleting these two items, the 19-item instrument has a two-factor structure based on Bandura's efficacy theory.

Efficacy Beliefs of Korean Preservice Teachers

Previous research indicated that US elementary preservice teachers' mathematics teaching efficacy beliefs increased during their teacher education program (Utley, Bryant, Moseley, 2005). However, the result of this study showed that the PMTE scores (sum of P-initial item scores) and MTOE scores (sum of O-initial items) are different between sophomores and seniors; the Korean elementary preservice teachers' mathematics teaching efficacy beliefs gradually decreased during the program. In the item-level analysis, participant responses were significantly different only for three items: Item 1 (*exerted a little extra effort*), Item 4 (*more effective teaching approach*), and Item 11 (*understand mathematics concepts*). These results imply that mathematics teaching efficacy of Korean elementary preservice teachers might not be changed very much.

Contextual and Cultural Influence

Culture can play a large role in determining teachers' efficacy beliefs (Lin, Gorrell, & Taylor, 2002). The changes in Korean preservice teachers' mathematics teaching efficacy beliefs in this study may be understood within the contextual and socio-cultural settings surrounding Korean elementary preservice teachers and the teacher education program.

The decrease of personal mathematics teaching efficacy of Korean elementary preservice teachers is possibly influenced by the burdensome course work in mathematics which requires understanding advanced mathematics. An elementary preservice teacher will not become a mathematics subject specialist but an all-subject generalist; understanding advanced mathematics is very demanding on them. As elementary preservice teachers move up to the next level, for example, from sophomore to junior, incomprehensive understanding of advanced mathematics is likely to accumulate. This academic challenge possibly leads to increased tension with their efficacy beliefs.

Excessive private education that is prevalent in Korean society is another potential factor that explains why preservice teachers' showed decreased outcome expectancy in mathematics teaching. In Korean culture, parental support for their children is highly emphasized as an important contributor to children's learning (Hwang, Lin, & Gorrell, 1999). Korean parents, today, push their children to take extra lessons from a private tutor and/or at a for-profit private academy. Many preservice teachers tutor a student in mathematics. Owing to such experience, they might have, for a moment, stronger feeling of personal teaching efficacy. However, they soon become aware that many students already know what they are expected to learn from their school teacher. Therefore, preservice teachers become aware that increasing a students' achievement will be very difficult, which perhaps lead to decreased outcome expectancy of preservice teachers.

Further Studies

In this study, possible factors to the statistical finding (decreased mathematics teaching efficacy) were suggested, but not investigated. Further study should test those longitudinal factors. Because the MTEBI was developed from a US sample, it may not be suitable for application in different cultures. Low reliability, alpha = .66, of the MTOE scale might indicate that Korean preservice teachers' outcome expectancy looks quite different from that of US preservice teachers. It is an interesting question how outcome expectancy is different between the US and Korean cultures. The sample size $N = 106$, despite the support of the literature, is possibly another concern. A future study will confirm the current findings in a larger sample collected at multiple universities.

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WHAT ARE UNDERGRADUATES LOOKING FOR IN A METHODS EXPERIENCE?

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To track the beliefs of teacher candidates in a secondary mathematics education program, sophomores, as well as seniors in a methods course, completed surveys. In general, the highest priorities of both groups included learning to write lessons, as well as how to focus on mathematical processes and how to manage a classroom. The lowest priority of both groups was participating in organizations that promote ongoing professional development. Most striking was that the beliefs held by both groups were similar, indicating little shifting of priorities as undergraduates progress through the program. Further study, which also includes interviews, is being conducted.

Working at a university that is accredited by the National Council for Accreditation of Teacher Education (NCATE), we are constantly evaluating and trying to improve the coursework and other experiences that are required of undergraduate teacher candidates. NCATE, working with the National Council of Teachers of Mathematics (NCTM), publishes three sets of standards – one each for the preparation of teachers at the elementary, middle, and secondary levels (see NCTM, 2003). In addition, documents such as *The Mathematical Preparation of Teachers* (CBMS, 2001) and NCTM's *Mathematics Teaching Today: Improving Practice, Improving Student Performance* (2007) have provided recommendations for the content and pedagogical competencies required for one to become an effective teacher of mathematics.

As the coordinator of the Adolescent-to-Young-Adult (AYA or Secondary) mathematics education program, I conducted a formal study of our program in the Spring Semester of 2010, which resulted in a final report with recommended programmatic changes. In the process of developing the report, I became interested in the differences (and similarities) between what we were offering in our undergraduate teacher preparation program at the secondary level when compared to the perceived needs of the teacher candidates. These candidates formally enter the program in the second year when they take a course entitled "Introduction to Secondary School Mathematics" (EDTL 2740). After taking more general methods and education courses in the third year, these students return to take the "Secondary Mathematics Teaching Methods" course

(EDTL 4740) in the fourth year, followed by student teaching and taking a mathematics education seminar during the final semester.

The sophomore-level introductory course was piloted and subsequently added to the program as a requirement for all students beginning in the 2009-2010 academic year. There were a number of reasons why the course was instituted. They are as follows:

- Students reported that they wanted a specific earlier exposure to the field of *mathematics* education, rather than waiting until the final year of college when they enrolled in a methods course.
- Students were often fearful of teaching assignments in the field, particularly when the assignment included teaching algebra or geometry courses that they themselves had not taken in eight years or more.
- Several additional needs surfaced that required more attention than the former course structures would allow, such as an introduction to technology used specifically for teaching mathematics (e.g., Geometer's Sketchpad, Fathom, Geogebra), and changes in standards and curriculum.

In short, one methods course in the senior year simply was not enough time to address all of the needs of a pre-service secondary mathematics teacher, so the new course was developed. The course description for the class is:

> Review of content typically taught in the secondary mathematics curriculum, including topics from algebra, geometry, trigonometry, statistics/probability, and discrete mathematics. An introduction to state and national Standards in mathematics, including mathematical process skills, inquiry through the use of hands on materials, and current instructional technology. Includes observations at a field site. Prerequisites: EDHD 2010 [Introduction to Teaching], "C" or higher in MATH 1310 [Calculus I] and at least 30 completed semester hours.

Much has been written about the belief structures of teacher candidates in mathematics education and how beliefs are transformed (or remain rigid) in their campus and field experiences. Deborah Ball (1990) pointed out that by the time an undergraduate reaches a teaching methods course, years of classroom experiences have already shaped the way the individual believes that mathematics teaching "should" be conducted. In a study of pre-service secondary mathematics teachers, the authors of another study concluded by stating that effects of teacher education programs will continue to be "random" until conclusions can be drawn about how specific activities affect the belief systems of new teachers (Cooney, Shealy, & Arvold, 1998). Ultimately, the beliefs held by teachers directly impact the decisions they make on a daily basis in the classroom (Schoenfeld, 1998). Consequently, an examination of the beliefs of teacher candidates in a mathematics education program can be helpful in determining activities and experiences that might serve to alter those beliefs and make them more effective teachers. Without these experiences, teachers will continue to "teach as they were taught," furthering the cycle of using ineffective instructional practices.

Methodology

In the study conducted during the Spring Semester of 2010 (described above), only seniors in mathematics education and graduates of the program were interviewed and surveyed, as well as being involved in focus group discussions. However, we were also interested in the beliefs and perceived needs of students in the early stages of the program. As a result, another follow-up study has begun, the initial results of which will be described in this paper.

The following are the research questions that were raised to guide the current study:

1. Which content and/or pedagogical issues are priorities (and least important) for students in the sophomore level pre-mathematics methods course (EDTL 2740)?
2. Which content and/or pedagogical issues are priorities (and least important) for students in the senior level mathematics methods course (EDTL 4740)?
3. Which priorities, if any, change as a teacher candidate progresses through the mathematics education program?

The answers to these questions, in turn, should lead to a more effective redesign of the program to better meet the needs of candidates.

In an attempt to answer these questions, a simple pilot study was conducted in the Fall Semester of 2010. A total of 47 teacher candidates – 26 in the pre-methods course (2740) and 21 in the methods course (4740) participated in a rank-ordering activity. Each student was given a list of 21 intended outcomes from the mathematics methods course with a blank next to each. The students were asked to read through the list and prioritize the items, with no ties, from 1 to 21, in order, where "1" means the "most important" issue, while "21" is the "least important issue" to learn about in the program. These 21 intended outcomes are listed in full in the Appendix. The results of this initial pilot will be described below.

Findings

Teacher candidates in the pre-methods course (most of whom were sophomores) selected the following five areas that they felt were most important in their preparation as teachers (see Table 1). The Statement column refers to the course objective as listed in the Appendix; the Score is the mean (average) of the rankings, where a lower average score indicates a higher priority; and the Description is a short summary of the content of the stated objective.

Table 1.
Top 5 Priorities of Pre-Methods Students

Statement	*Score* (n = 26)	*Description*
12	3.41	Essential components of a lesson plan
3	5.15	How to make mathematical processes a focus of teaching
16	5.85	Promote positive classroom management
15	6.52	Meeting individual needs with technology and manipulatives
9	6.78	Questioning strategies to promote discourse

In terms of what they consider to be the lowest priorities of a mathematics education program, pre-methods students reported the following (Table 2):

Table 2.
Lowest 5 Priorities of Pre-Methods Students

Statement	*Score* (n = 26)	*Description*
21	16.44	Participate in programs and organizations that promote ongoing professional development
18	15.52	Explore ways that teachers can gather teaching ideas, including electronic sources
6	15.22	Explore research on appropriate use of technology in mathematics instruction
8	14.56	Identify, select, and use software and hardware that is appropriate for the classroom
19	14.07	Grow in appreciation for the role of mathematics in early adolescent education

The main concerns of students who are early in the secondary mathematics education program were in the areas of lesson planning, focusing students on mathematical processes, and managing their classrooms. On the other hand, their lowest priorities were on learning about and participating in professional development activities, exploring ways to find teaching ideas, and learning about how technology can be used in mathematics instruction.

When teacher candidates enrolled in a methods course (all seniors) were surveyed, they selected the exact same five priorities as the pre-methods students, as shown in Table 3.

Table 3.

Top 5 Priorities of Senior Methods Students

Statement	*Score* (n = 21)	*Description*
12	4.10	Essential components of a lesson plan
9	6.19	Questioning strategies to promote discourse
15	6.19	Meeting individual needs with technology and manipulatives
3	7.00	How to make mathematical processes a focus of teaching
16	7.33	Promote positive classroom management

While methods students selected the same five priorities, the order of choice was slightly different, with a higher priority placed on questioning strategies for the classroom but a lower priority on how to incorporate mathematical process skills into their teaching. The methods students also chose five areas that they considered to be their lowest priorities, which are shown in Table 4.

Table 4.

Lowest 5 Priorities of Senior Methods Students

Statement	*Score* (n = 21)	*Description*
21	17.62	Participate in programs and organizations that promote ongoing professional development
20	16.76	Continue to develop a positive disposition toward the study of mathematics
6	16.62	Explore research on appropriate use of technology in mathematics instruction
5	16.14	Describe popular learning theories on how students learn mathematics
19	14.29	Grow in appreciation for the role of mathematics in early adolescent education

Again, there are similarities and differences between the two groups in terms of lowest priorities. For both groups, participation in professional development activities was rated the lowest of the

21 statements. But the second lowest choice for methods students was on development of a positive disposition, while the same statement rated 15 out of 21 for pre-methods students. Similarly, while pre-methods students rated statement #8 dealing with selection and use of hardware and software in the classroom as their fourth lowest priority, methods students chose technology as 13th out of 21 statements.

Analysis and Comments

One of the most striking surprises of this pilot survey was that the top five priorities of pre-methods and methods teacher candidates were identical, with relative positions being slightly shifted. While one might expect a pre-methods student to be primarily interested in the field of mathematics education, where teachers search for ideas, and how to obtain professional development, the survey shows that this group of teacher candidates has as much interest in how to develop lesson plans and manage a classroom as the seniors in the methods course. Also, while a major emphasis of both of the courses is on professional development – with teacher candidates frequently hearing about how "one does not become an effective mathematics teacher with only four years of undergraduate education" – both groups believe that participation in professional development activities and exposure to organizations such as the NCTM and its affiliates are their lowest priorities. Therefore, in terms of the third research question regarding how priorities evolve as students progress through the program, the results of this pilot study indicate that beliefs do not change very much at all between the sophomore and senior years.

The current study raises a number of new questions that can be pursued further, such as:

1. Will these same priorities surface in future semesters, across groups?
2. Why do the priorities of teacher candidates remain relatively "fixed" over a three-year period?
3. Can any significant differences among priorities between pre-methods and methods teacher candidates be identified after collecting data from another semester or two?
4. How can faculty and program redesign begin to address these attitudes? For example, the university feels strongly about developing teachers into individuals who seek ongoing growth and professional development activities, yet teacher candidates do not place this as a priority. What can be done in the program to instill this value in new teachers?
5. What additional research can be conducted to assess and track teacher candidate values on the course syllabus?

In an effort to address these questions, a more fully-developed survey has been designed and approved by the university's Human Subjects Review Board for the Spring Semester of 2011, including the use of randomly-selected structured interview follow-ups to find out why teacher candidates reply the way they do. For example, asking an individual the direct question, "Why did you rate statement #21 as your lowest priority?" may help researchers to rethink the role of professional development in the program. Similarly, asking a pre-methods student why he/she placed such a high priority on lesson planning, when the program does not require him/her to write a plan for at least another year would shed additional light on the fixed nature of planning as a priority.

Teacher education programs are designed not only to strengthen content knowledge but also to influence beliefs and attitudes of candidates. Research such as this pilot study and its follow-up is necessary to track candidate opinions and to strategize for how to best influence their attitudes over time. BGSU, for example, is one of only three universities in the State of Ohio that has a student mathematics education organization that is affiliated with the NCTM. Yet if teacher candidates do not view professional development as a priority, the organization will never reach its potential. Therefore, continuing to gather and analyze data about student beliefs in the mathematics education program is essential to enhancing the program and, ultimately, providing the best possible preparation for candidates.

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Appendix

1. Describe the significance and general content of the Standards documents of the National Council of Teachers of Mathematics – *Principles and Standards for School Mathematics, Professional Standards for Teaching and Learning Mathematics*, and *Assessment Standards for School Mathematics*.
2. Demonstrate an understanding of the philosophy and content of the Ohio Academic Content Standards and its impact on reform in mathematics education in Ohio (including Achievement testing and the High School Graduation Test).
3. Describe (and illustrate in lesson planning) how to make the five mathematical processes – problem solving, reasoning and proof, communication, connections, and representation – the focus of an adolescent/young adult mathematics program.
4. Compare/contrast various curricular sequencing models for mathematics instruction, including the traditional Algebra-Geometry-Algebra II program and an Integrated approach.
5. Describe popular learning theories that attempt to explain how students learn mathematics and computer science, including comparison and contrasting of the theories of Piaget (the constructivist viewpoint), Vygotsky, the Van Hieles, and Bruner.
6. Explain how research in mathematics and technology education is conducted, reported, and applied to reform in teaching and learning practices, with an emphasis on differentiating between appropriate and inappropriate use of technology.
7. Illustrate how to use technology (e.g., graphing calculators, computer software, video programs, CD-ROMs, and the Internet) and identify the benefits of technology to maximize student learning.
8. Identify, select, and use hardware and software technology resources to meet specific teaching and learning objectives.
9. Give examples of questioning strategies for the classroom that promote mathematical thinking and dialogue (discourse).
10. Use cooperative learning strategies in mathematics instruction.
11. Write instructional objectives at the knowledge/skill, conceptual, and application levels.
12. Recognize the essential components of a lesson plan and prepare a mathematics lesson plan which includes outcomes, materials, a motivating activity, a structured sequence of experiences for the students, a logical closure, a planned extension, and a plan for assessment.
13. Recognize the use of technology-enriched learning activities in the classroom and write lesson plans that make use of technology to address diverse student needs, as appropriate and available.
14. Prepare short-range and long-range unit plans that illustrate connections between lessons.
15. Recognize that each student has individual needs and illustrate how a variety of teaching approaches, including the use of manipulatives and the use of technology, can be used to appeal to the learning style of each student.
16. Describe a variety of strategies that teachers can use to promote positive classroom management and the role that effective lesson planning has on classroom environment.
17. Illustrate the ability to use a variety of assessment strategies to collect data, including electronic means, regarding student academic progress and the development of dispositions towards mathematics.

18. Explore a variety of ways in which teachers can gather field tested ideas for use in one's own classroom, including electronic sources.
19. Grow in his/her appreciation of the role of mathematics in the adolescent/young adult curriculum.
20. Continue to develop a positive disposition toward the field of mathematics.
21. Become familiar with and participate in programs provided for continued professional growth in the field of mathematics education, including the NCTM, OCTM, BGCTM, etc., including by means of the Internet and other electronic sources.

TRANSFORMING PERCEPTIONS OF MATHEMATICAL KNOWLEDGE FOR AND THROUGH SOCIAL UNDERSTANDING

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Preservice teachers often enter college mathematics courses with limited conceptual understanding of mathematics. This study examines what happened to a group of prospective teachers' perceptions of their knowledge of mathematics when it was integrated with social issues in a content course for preservice teachers. It also examined these participants' perceptions of their knowledge of social issues and the relationship between mathematics and social issues.

With global problems arguably at their worst in history (Bender, Burns, Burns, & Guggenheim, 2006), in recent decades, teachers and scholars have begun addressing the idea of teaching mathematics and social issues in more integrated ways (e.g., Powell & Frankenstein, 1997; Gutstein & Peterson, 2006). Although some mathematics educators have begun exploring the consequences of teaching mathematics alongside social issues with prospective teachers (e.g., Spielman, 2009), not many depictions of these students' perceptions of learning mathematics in this way exist. Therefore, in this study I examine one group of preservice elementary teachers' perceptions of their knowledge of mathematics, social issues, and the relationship therein when mathematics and social issues are integrated in a mathematics content course for elementary teachers.

Theoretical Lenses

I draw on two theoretical lenses to explain the findings of this study. The first explores the idea of applicable subject matter. The thought is that both mathematics and social issues are more meaningfully understood when they intertwine with one another (Gutstein & Peterson, 2006). Throughout the past century, scholars such as Dewey (1902) and Whitehead (1927) have called into question the abstract nature of teaching. They have contended that the primary goal of education should be connecting education to the lives of students, not transmitting disconnected, seemingly irrelevant facts from a more knowing authority to a less knowing pupil. Therefore, in theory, if preservice teachers see meaningful connections between mathematics and the world around them, they are more likely to learn the material and use it in the practical.

The second lens I utilize to explicate the findings is the idea that knowledge must be actively constructed and socially verified. Constructivist theories of learning suggest that listening to a perceived authority is insufficient for the development of knowledge (von Glasersfeld, 1995). Students must experience mathematics in multiple and individualized ways to form connections and develop their own conceptual understandings. Therefore, if preservice teachers engage in activities and experiences that support problem-centered learning approaches to teaching and learning mathematics (Wheatley & Abshire, 2002; Van de Wall, 2004), theoretically, they will actively construct deeper and more conceptual understandings of mathematics.

Methodology

Data for the study were collected in a class I taught at a community college in a city in the Southwestern region of the United States. All nineteen students in the course were female, ranging in age from eighteen to forty-three. All of the students had at least one pre-requisite college mathematics course before entering this class or had passed a placement exam for entrance into the course. Seven of the students were English language learners.

There were two objectives for the course, a primary mathematical one and a secondary social one. The mathematical objective of the course included developing mathematical number sense through problem solving, critical questioning of traditional mathematics instruction, and the integration of social issues. I, the instructor, deliberately selected activities meant to engage students in developing conceptual understandings of mathematics and finding connections between mathematics and meaningful contexts. The departmental curricular objectives for the course focused on number, arithmetic operations, fractions, decimals, percentages, operations with proportions, sets, and operations with integers. The social objectives I supplemented focused on social growth, or the development of the disposition and mathematical skills necessary to understand critical social issues and participate more comprehensively in decision making in a democratic society. Social issues such as healthcare reform, the ethics of sweatshops, television advertising, and poverty were integrated into the curriculum throughout the semester, often times with the aid of the arts (i.e., children's stories, newspaper articles, video clips, etc.).

The study utilized a qualitative research design that incorporated case study (Stake, 1995) and practitioner-research (Anderson, Herr, & Nihlen, 1994). Data were collected in the forms of a reflective journal I completed weekly, student work (which included written reflections), recorded classroom discussions, and recorded informal interviews. Upon completing data collection, I used a data analysis spiral (Cresswell, 2007) approach to find analytical themes. Reflective examination of data occurred throughout the semester; however, formal data analysis included the development of an overall picture of student perceptions through successive examinations of my journal, student journals and audio/video recordings. Audio/video recordings were transcribed, and several spirals back through the data were performed to verify the formation of themes from which I identified answers to the research question.

Findings

Analysis of the data revealed major findings about the transformation of these prospective teachers' knowledge of both mathematics and social issues when they were intertwined in a problem-centered learning environment.

Initial Knowledge

Data revealed that meaningful knowledge of both mathematics and social issues was initially limited. In accordance with other mathematics educators' findings, data in this class indicated that as students spoke and wrote about mathematics in the first few weeks of the course, they seemed to exhibit knowledge that existed in bits and pieces, often consisting of claims that could not be substantially supported or were just incorrect (Young, 2002). However, these findings were not only applicable to mathematical knowledge but also to social issues knowledge. Further, participants in this study seemed to view mathematical topics and social issues as unrelated, both from within and across the two subject areas.

Limited Knowledge. Some students recognized limited mathematical knowledge in themselves. For example, during a discussion of fractions, one student explained that she always "skipped those problems" when she encountered them on exams or in homework and another student said, "I never learned fractions when we did them in school." In one informal interview, a student said:

> Percentages are clueless to me. I wish I could learn them, because I go to the store and it will say "20% off" and I'm like "man, okay, how much is that?" I hate it when I can't figure it out.

Similarly, some recognized their limited knowledge of social issues. They revealed that they "don't follow" or are confused by many of the social issues we encountered. For example, students wrote:

> Personally, I don't follow what President Obama is doing… (Student journal)
>
> As an international student, I really had trouble to understand what is the current health insurance system in the United States. Besides, the idea of health care reform made me more puzzled… (Student journal)
>
> I do not follow politics very much… (Student journal)

Others discovered their limited knowledge of mathematics as they began to struggle with unfamiliar concepts. For example, as we studied the history of mathematics, students were asked to perform operations on numbers in bases other than base ten. They were instructed to find the following sums:

$$304_{\text{five}} + 20_{\text{five}} + 120_{\text{five}} + 22_{\text{five}} =$$

$$201_{\text{three}} + 102_{\text{three}} =$$

Many students proposed the answers as:

$$304_{\text{five}} + 20_{\text{five}} + 120_{\text{five}} + 22_{\text{five}} = 466_{\text{five}} \text{ or } 121_{\text{five}}$$

$$201_{\text{three}} + 102_{\text{three}} = 303_{\text{three}}$$

Correct Answers:

$$304_{\text{five}} + 20_{\text{five}} + 120_{\text{five}} + 22_{\text{five}} = 1021_{\text{five}}$$

$$201_{\text{three}} + 102_{\text{three}} = 1010_{\text{three}}$$

They had difficulty recognizing when to switch from one digit's place value to the next, and even when they did, they often forgot to place a zero in the number if one digit's place value was not a number greater than zero (as is indicated by the second incorrect solution to the base-five problem). This recognition of limited understanding seemed to frustrate students as they discussed these problems in class.

Similarly, students discovered their limited knowledge of some social issues. During a discussion of poverty in our state, students discovered that approximately sixteen percent of the population lives in poverty. I wrote in my reflective journal that students seemed surprised by the figures. In another activity, while students learned about sweatshop worker wages and conditions, they realized that many sweatshops exist in the United States. Students made comments like, "I didn't know we have sweatshops here. I thought they were only in other countries." As students reflected on the lessons we did, many wrote about being astonished by the limited knowledge people have of critical social issues. One student wrote, "I learned how little most people truly know about very important subjects."

Further, many students saw mathematical concepts as disconnected from one another and initially never spoke of connections to situations outside of mathematics. For example, during a discussion of multiplication of fractions, after letting students grapple unsuccessfully with explanations as to why we multiply the tops and the bottoms, I asked students to describe how they thought about multiplication of whole numbers. The discussion of this topic formed a connection for one student which prompted her to say, "I had no idea you could think of multiplication of fractions in the same way." Several students affirmed this statement. During another discussion, I asked students to find 20% of the United States' population. One student said she divided by five to do this, another student commented that she used a formula for finding 20%, a third student explained that she found the figure by finding 10% and doubling it, and a fourth student said she multiplied by .2. The various ways of finding 20% yielded a long discussion about the similarities between percentages, decimals, division and benchmarks. Many students expressed their surprise by these links.

In a similar fashion, students pigeonholed their knowledge of social concepts, viewing different contexts as disconnected from one another. For example, students used words such as socialism, oppression, and dictator together and words such as capitalism, freedom, and democracy together, portraying these groups as disjointed and unrelated to one another. They seemed to have difficulty envisioning overlap, as was illustrated when a discussion of socialism led to the idea that socialism exists in the United States. Students gave the impression that they were surprised to learn that the fire department, the police department, public schools and libraries are examples of socialism. They appeared to have difficulty envisioning that there are instances when concepts such as socialism can be good or can have anything to do with democracy or the United States.

Unsupported Knowledge. For several students, superficial mathematical knowledge seemed to exist, but abilities to conceptually support the mathematical operations they performed were bounded. For example, during a discussion of multiplication of integers, almost every student could supply the rules for multiplying positive and negative numbers, but not one could explain the logic of all the rules. Most students defended procedures with statements such as, “that’s the rule” or “because my teacher said so,” and one student wrote in her journal:

> I’ve never thought about the mathematical fundamentals which the formula derived from real examples, but just memorized formula, substitute some numbers for X or Y in it, and did the mechanical calculations. (Student journal)

During a discussion about multiplication of multiple-digit numbers, one student said, “I thought I understood how to multiply a two-digit number with another two-digit number, but it never occurred to me why we scoot over on the second line.” She had always used the procedure but never thought about the logic of performing the calculation in this way.

Although most students had some exposure to the social issues that emerged in this class, just as with the mathematics they explored, their knowledge was often unsupported. For example, in my reflective journal I noted one student objecting to health care reform “because doctors would make less money, and no one will want to be a doctor.” When I asked her how much doctors would make, she answered, “I don’t know, thirty or forty thousand.” She believed she had a solid argument, but it was limited to a superficial statement, not a supported understanding.

Moreover, some students illustrated their inability to support their knowledge by using one or two examples as substantial evidence of an argument. For example, one student wrote in her journal:

> Doctors don’t make money through Medicaid or Medicare…when I was 18 years I used to work with a Dr and his wife used to put things in the forms that was not even done to the patients so they could get more money. By the time I retired and want to get Medicare, there is not going to be any… (Student journal)

Students based their knowledge on belief rather than evidence. On several occasions, I noted in my reflective journal that students would use phrases such as “I think,” “I believe,” or “In my opinion” to describe their perspectives of issues we came across. One student described poverty as a problem in our state because, as she put it, “I think people are just lazy and don’t wanna’ get a job.” Another student objected to this statement and explained that she believed that a lot of those people “can’t get a job.” She too, explained this as her opinion. They did not seem to find it necessary to verify their claims with supportive data.

Erroneous Knowledge. Throughout the semester, students often revealed their constructions of erroneous mathematical and social issue knowledge. For example, during a discussion of operations with fractions, I asked students to tell me what they knew about multiplication of fractions. One student said, “Don’t you need a common denominator?” She was using the rule for addition of fractions and applying it to multiplication of fractions. Yet on another occasion, a student proposed that for addition of integers, “when the signs are different, your answer is negative and when the signs are the same, your answer is positive,” applying the rule for multiplication of integers to addition of integers.

Likewise, some students seemed to have inaccurate knowledge of social issues. For example, before a mathematical healthcare debate, one student wrote in her journal, “I don’t find it right for me to pay for someone else’s healthcare.” She had explained that her opposition to healthcare reform stemmed from this reason. She seemed not to recognize that she pays for uninsured citizens’ healthcare under the current system. Further, several students agreed that they opposed healthcare reform because, as one student put it, “People with socialized medicine pay more for healthcare.” However, during the healthcare debate, after conducting their own research, students discovered that Americans pay more for healthcare per capita than any other industrialized country in the world.

Transformed Knowledge

As students interacted with one another, the instructor, and the curriculum, they began to transform their knowledge of both mathematics and social issues. They initiated discussions and wrote about mathematics and social issues with more clarity and depth, supporting their mathematical ideas with conceptual reasoning and defending their claims of both mathematics and social issues with evidence. Although other mathematics educators have found that when students actively and socially construct mathematical knowledge, they begin to view the subject in more meaningful ways, they begin to defend their reasoning, they begin to see connections between mathematical topics, and they begin to represent mathematics in multiple ways (e.g., Young, 2002), I found that social issue knowledge transformed in a similar fashion when it was intertwined with mathematics. By the end of the semester, many students not only started to see connections among mathematical content and social issues but also between the subjects, forming connections between mathematics and a world outside of the classroom. By the last day of class every student had either written or spoken about how this course helped transform her understandings of both mathematics and social issues.

Meaningful Knowledge. Data revealed that the most significant transformation for students was constructing meaningful mathematical and social issue knowledge. As the semester progressed, students began to speak and write about their transformed knowledge with more depth and an enthusiasm that seemed to not exist previously. As for their mathematical understanding, many students wrote about how engaging in this course increased conceptual comprehension for them. One student wrote:

> I definitely feel I learned a lot from this class. Mostly, I have learned the concepts behind the math problems that I've been doing my whole life which I thought was very important, and I realized that the way I had been taught was kind of sad in a way! I was taught repetitive procedures and had no clue as to what was behind the concept… (Student journal)

During a discussion about fractions, one student said, "I've been through elementary school, middle school, high school, and two college math classes, and I never understood fractions the way I understood them in this class."

Many students came to this class with definite opinions about social issues but could not discuss those ideas with clarity and depth. However, after exploring the issues through mathematics, most of the students in the class began writing about the issues with much more understanding. As students reflected on the healthcare debate in their journals, one student wrote:

> The U.S. spends far more than any other industrialized nation on healthcare. Yet, other nations insure everyone while America has 46 million uninsured, a number which will grow as health insurance costs rise… (Student journal)

Opinions about healthcare reform began to shift from opposition to favor. By the end of the debate, the more wholistic understanding of this social issue persuaded many students to not only support healthcare reform but also favor a universal healthcare system. Students wrote:

> According to statistics from 2003, the United States spends $5,711 per capita per year for health care while Canada spends about half of that, $2,998 per capita per year (Kaiser Family Foundation, 2007)… socialized healthcare does work. (Student journal)

> I learned a lot of great statistics from the debate. I learned that socialized healthcare would benefit more people than it would hurt. ..Socialized healthcare is established in many countries…This program is working perfectly fine in these countries. (Student journal)

They began forming judgments about mathematics and social issues based on a more explicit understanding of issues through a conceptual understanding of mathematics rather than simply relying on emotion, procedure, or opinion.

Supported Knowledge. As students began to construct more meaningful and explicit understandings of both mathematics and social issues, they began to gain knowledge of how to support their own reasoning. They learned to refrain from asking for "the teacher's" input as they realized that I would not supply them with hints, solutions, or opinions; and they began to search for explicit understanding from within. They wrote:

> I am gaining confidence and an increase in familiarity with problem solving and hopefully logic and reasoning… (Student journal)
>
> The main thing that impacted me was your teaching style. I really hope to be able to encourage my students to learn by letting them figure it out on their own like you did with us… (Student journal)
>
> I think the thing I like the most about it [the class] is being able to figure out the problems…A lot of times it takes me a little bit of extra work to figure things out. But that['s] okay. (Student journal)

As students' understanding and confidence grew, they began to not only support their own mathematical reasoning but also their newfound social ideas. During the classroom debate, one student supplied a figure for the cost of healthcare reform on families in the United States. Another student quickly referenced a different figure that showed a lowering of cost for families in the U.S. and she explained why this would be the case.

Students began to question one another's knowledge and ask for evidence to claims others would make. In my journal, I wrote that after students engaged in a couple of social issue lessons, they no longer took classmates' comments "at face value." I wrote about one student making a comment regarding sweatshops and another saying, "Where did you get that information, because I found something different?" They began to view a meaningful understanding of social issues as based on evidence rather than unsupported statements.

Students even became more critical of their own understandings. One student wrote about herself, "To be honest I did not know a thing about this healthcare reform…This is bad for me because I should be informed." The recognition of their limited knowledge convinced some students to advocate teaching for social understanding. Students wrote:

> Incorporating math into those everyday things is so important, especially for kids because we should be teaching them to become better PEOPLE, not just better STUDENTS!! (Student journal)

> We reclaim society from giving attention, rediscovering on many controversial social issues. Throughout this process we can find possible answers. Teachers are not people who hand down only scholastic knowledge to the next generation, but also help them to build desirable insight into our social problems… (Student journal)

Wholistic Knowledge. When students began to support claims and understand mathematics and social issues in more meaningful ways, they began to form connections that expanded notions of what it means to solve mathematics problems or comprehend social issues. They began to develop new ways of constructing knowledge—ways that questioned traditional assumptions of solving problems in one way, ways that related mathematics and social issues to each other, and ways that prompted questions beyond the scope of the objectives for the course. Developing knowledge became dynamic and unrestricted to single procedures. Social issues became mathematical, and mathematics became a social issue. Student knowledge began to transcend traditional boundaries of attaining correct answers that exist in isolated subject areas.

The curricular routines of this class allowed students to find mathematical connections using various methods of solving problems. Students began using manipulatives, drawing pictures, and relating topics to previously studied concepts or ideas. In their journals, students wrote about the impact of solving mathematics in various ways:

> I learned that there isn't just one way to solve a math problem. You don't always have to remember the "rules". You can use base ten, fraction bars and other manipulative kits… I definitely learned more about fractions! Thinking of it as multiplication and benchmarks helped…

During a discussion of decimals, one student said, "I never thought to use base-ten blocks when working with decimals, but they really help." Students even began to understand important relationships between mathematics and social issues. They wrote:

> As a student of the math class, I realized that we are using "MATH" a lot in our real life, not only calculating for our receipt in a store but also reading what happens in our community. Statistic and many kinds of graph can convey a whole story… (Student journal)

> These lessons made me think outside of my personal box. Additionally, the lessons showed me just how important math is in our daily lives. (Student journal)

These connections often led students to further questions. For example, while studying sets and whole numbers, one student noticed that every time she would subtract two odd numbers, she would obtain an even number. She asked me if I thought that would always be the case. I redirected the question to the other students, and by the end of the class, a student illustrated a proof of the conjecture that "an odd number minus an odd number equals an even number." In another class period, I observed a student noticing a pattern when working with exponential growth. She proposed the pattern to the class, and we worked together to verify it. It was an unexpected result that led to a discussion of summation notation and how finding a pattern can lead to writing a formula. Further, in their reflective journals, students wrote about their newfound inquiries of social issues:

> The lessons on social issues I think helped the class not only to incorporate and think about the math within the subject, but sparked further interest in the issue itself… (Student journal)
>
> I found that I would continue to think about these issues days and weeks after the lesson had concluded… (Student journal)
>
> I learned that I have many more questions economical, fiscal, financial and political. (Student journal)

Engaging these students in mathematics linked with social issues deepened their understandings of both mathematics and social issues, helped them formulate and defend their knowledge, and began to bridge the divide that existed for many of them between mathematics and the world outside the classroom. Further, engaging these prospective teachers in this type of integrated learning sparked an interest in the mathematics and the issues, encouraging students to continue to explore them, even when we were not deliberately addressing them in class. These preservice teachers began to value a meaningful understanding of mathematics and social issues and some even started advocating educating for social well-being.

Implications for Mathematics Education

The data in this study illustrated not only the mathematical but also the social growth the prospective elementary teachers in this course experienced from the connections of mathematics to social issues in a problem-centered learning environment. This study suggests that mathematical and social understanding can be meaningfully integrated in a mathematics content course for elementary teachers. However, integrating mathematics and social issues does not make success inevitable. The examples presented in this paper do no not encompass the totality of why most students succeeded in this course or the limitations and resistance encountered with this approach to teaching. I recognize that rarely are experiences in any class as simplistic as they are often presented in papers such as this. The complexity of classroom interactions interwoven with instructor relationships with students, the rapport developed between students, and the social norms established, all play critical roles in the success of any teaching endeavor. Although I believe my results are accurate, many factors of the class could not be described in this paper. Therefore, it is important to note that maximizing mathematical and social understanding is not limited to incorporating social issues into the curriculum, but rather that doing so can aid in this process.

With this in mind, the findings suggest that the college students in this study responded well to the incorporation of social issues into this mathematics class. Consistent with many scholars' (Dewey, 1902; Gutstein & Peterson, 2006; von Glasersfeld, 1995; Whitehead, 1929) suggestion that education is better understood when it is relevant, actively constructed, and socially verified, I found that connecting mathematics to social issues in this problem-centered learning environment supported these presrevice teachers' interest in and understandings of both mathematics and social issues. The findings of this study imply that when social issues and problem-centered learning are incorporated in meaningful ways, students can become motivated to learn both mathematics and social issues on their own. Therefore, I would advocate creating a space where creativity in mathematics can emerge by centering class time on relevant discussions. However, I would also emphasize that this is only one component to a successful relationship between the teacher, the curriculum, and the students.

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A CONCEPTUAL APPROACH TO FUNCTION TRANSFORMATIONS

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Transformations of functions is a topic that is often taught by memorizing and applying rules without an understanding of the underlying concept. A two-week instructional unit using the recommendations of Faulkenberry and Faulkenberry (2010), which describes a method of teaching transformations of functions in a conceptual way, was implemented. Pre-test and post-test results showed significant gains in students' conceptual understanding of transformations of functions.

Research has established the importance of understanding a concept in order to become proficient in a subject. When students understand a topic, they are able to apply their knowledge flexibly to new situations. The goal of teaching mathematics is for students to develop mathematical understanding. That is, the students should acquire knowledge of mathematical concepts and procedures, the relationships among them, and why the procedures work (NCTM, 2000). Students who memorize facts or procedures without understanding are often not sure when or how to apply their knowledge (Ma, 1999). Thus, current reform movements in mathematics education call for an increased attention to conceptual understanding of topics rather than rote memorization of tasks and procedures (NCTM, 2000, 2006).

Conceptual knowledge refers to knowledge of the underlying structure of mathematics – the relationships and interconnections of ideas that explain and give meaning to mathematical procedures (Rittle-Johnson & Alibali, 1999). Conceptual knowledge goes beyond knowing two concepts are connected to knowing "how" they are connected. A lack of conceptual knowledge can result in incorrect application of procedures and misconceptions (Hiebert & Lefevre, 1986; Skemp, 1976). Students are more likely to understand why a procedure works if that understanding is established before the students gain a routinized understanding of how the procedure works (Graeber & Tanenhaus, 1993).

Transformations of functions is a topic that is often taught by memorizing and applying rules without an understanding of the underlying concept. The National Mathematics Advisory Panel (2008) report states that many students do not understand the procedures for transforming functions or why they are done the way they are. The National Council of Teachers of

Mathematics (NCTM) emphasizes relationships and the analysis of change in the study of functions (2000). They also recommend using transformations to analyze mathematical situations. The *Curriculum and Evaluation Standards for School Mathematics* (1989) state that the curriculum should focus on a transformational approach to graphing functions rather than graphing using a table of values. Using such an approach highlights the connections between geometry and algebra and provides a structure through which the students can explore properties of functions (NCTM, 1989).

In Faulkenberry and Faulkenberry (2010), the authors describe a method of teaching transformations of functions in a conceptual way. This method includes focusing on how changes in the input and output values change the resulting function. These graphical changes can be visualized using graph paper and transparencies. Changes in output manifest themselves as vertical changes on the graph. Changes in input manifest themselves as horizontal changes on the graph. Changes in output occur after the function has been applied to the input value and affect the y-values of the graph, therefore a transformation of increases the y-value by *d* which is exhibited as a shift up of *d* units. Changes in input occur before the function is applied and so must be applied to all values in the domain of the function. The domain of the function can be visualized on the *x*-axis therefore any changes to the input are exhibited by shifting the *x*-axis. Thus a transformation of can be visualized as moving the *x*-axis *c* units to the right. This movement of the axis to the right appears on the graph as a shift of *c* units to the left. This apparent movement can be easily shown using graph paper representing the axes and a transparency with the graph of *f* on it. To show output changes, one moves the transparency with the graph. To show input changes, one moves the graph paper with the axes.

Multiplicative transformations work similarly. A transformation of the type that changes output values and so can be visualized as a vertical stretch or compression of the graph of the function. A transformation of the type $y = f(bx)$ changes the input values and can be thought of as multiplying each input value by *b*. This can be visualized as stretching the *x*-axis like a rubber band. When the *x*-axis snaps back into place, the graph of *f* appears to be compressed by a factor of *b*.

Given that this method of instruction is a novel approach that views the concepts of input and output independently, and focuses on how changes in input and output affect the function, we wanted to determine if this approach is effective in promoting gains in knowledge about function transformations.

Method

Participants

Twenty-six undergraduate pre-service teachers (24 female) enrolled in a mathematics course for middle school mathematics teachers at Texas A&M University-Commerce participated in the study. These students were juniors and seniors who are pursuing middle school mathematics teacher certification.

Materials and Procedure

A paper test containing items reflecting knowledge of transformations of functions was administered before and after an instructional unit on transformations of functions. The test (see Appendix) contained items in three main groups: graphical representations, connecting symbolic representations to physical motion, and numerical representations. There were three items testing graphical representations, three items testing the connection of symbolic representations to physical motion, and four items reflecting transformations of table based functions.

The two-week, inquiry-based instruction phase used the recommendations of Faulkenberry and Faulkenberry (2010) to connect function transformations to changes in input and output values. The instruction began with a demonstration of additive transformations of functions using graph paper and a transparency focusing on the motion of the graph resulting from changes to input and output, described in the referenced article. This was followed by a teacher-led, whole-class discussion of multiplicative changes and how they stretch or compress the axes and how these stretches and compressions affect graphical motion. The instruction concluded with an exploration of tabular representations of functions and how various changes in input and output affect the table values. A summarizing activity designed to connect the tabular changes with symbolic representations and graphical motion concluded the instructional phase.

Results and Discussion

Overall Performance

There were 26 possible points on the paper test. There were 6 points possible (2 points for each graph) for the graphical representations, 6 points possible (2 points for each equation) for the symbolic representations, and 14 points possible (1/2 point for each value) for the tabular representation. A summary of the performance data can be found in Table 1.

Table 1

Mean scores by topic (standard deviation in parentheses

	Graphical Representations	Motions from Symbols	Tabular Transformations	Overall
Pre-test	1.1 (1.3)	0.6 (0.9)	1.6 (2.8)	3.3 (3.8)
Post-test	4.6 (1.4)	4.6 (1.7)	10.7 (5.1)	19.9 (6.0)
Gain	3.5***	4.0***	9.1***	16.6***

Note: ***p<0.001

Participants made significant gains in their overall test score from pretest to posttest. At pretest, participants scored a mean of 3.3 (12.7% of possible points). At posttest, the participants scored a mean of 19.9 (76.5% of possible points). This represented a mean gain of 16.6 points ($t(25) = 13.29$, $p < 0.001$). This gain is remarkable in that it represents a 503% increase from the pretest. With regard to graphical representations of transformations of functions, participants significantly improved from a mean score of 1.1 on the pretest to a mean score of 4.6 on the posttest ($t(25) = 8.57$, $p < 0.001$). Similarly, participants made significant gains in their understanding of how symbolic changes affect motion by improving from a mean score of 0.6 on the pretest to a mean score of 4.6 on the posttest ($t(25) = 11.55$, $p < 0.001$). Likewise, significant improvements were made with regard to numerical representations of transformations of functions with a 9.1 point gain from a mean of 1.6 on the pretest to a mean of 10.7 on the posttest ($t(25) = 9.11$, $p < 0.001$).

In addition, analysis of the individual test forms indicates conceptual understanding of the topic with the use of phrases such as “the grid moves two places left which means the graph moves two places to the right” and “it shrinks the grid which in return stretches the graph.” The students were able to classify transformations based on whether the input or output was being modified. The participants also described the transformations as vertical or horizontal. During the instructional unit, the students made comments such as “This makes so much sense!” and “Why didn’t they teach us this before?”

These results are in accord with two main theoretical perspectives in mathematical knowledge development; the concepts-first view (Byrnes & Wasik, 1991) and the iterative concept/procedures view (Rittle-Johnson, Siegler, & Alibali, 2001). In the concepts-first framework, mathematical procedures are learned best when prefaced with conceptual development. If procedures are taught without conceptual understanding, they tend to be prone to “bugs” (Brown & Van Lehn, 1982; Skemp, 1976). This framework is supported by the current study. By employing a conceptual approach to teaching function transformations, students were able to successfully develop procedural rules for approaching such problems. This is especially clear in the case of tabular function representations, as this is a type of function representation that is rarely covered in college-level algebra courses. However, the students were able to devise “rules” for filling out the table without being explicitly taught.

In the iterative framework (Rittle-Johnson, Siegler, & Alibali, 2001), concepts and procedures are mutually reinforcing. Concepts can help students learn why procedures work, but on the other hand, knowing procedures can provide motivation for learning why the procedures work. This view is also supported by the current work. Many students reported already knowing procedures (albeit “buggy” ones) for graphing transformations of functions. As such, the students were especially interested in finding out why the rules worked the way they did. In this sense, the knowledge of procedures likely provided the necessary motivation for learning the concepts behind function transformations.

One limitation of the current study is the lack of a comparison group. Indeed, the results would be even stronger if they showed positive gains above and beyond the gains of a comparison group that received a traditional unit of instruction. This would be an excellent future study.

Conclusion

In conclusion, the students showed significant gains in their knowledge of transformations of functions following the instructional unit based on Faulkenberry and Faulkenberry (2010). Not only were they able to describe how a function would move based on certain transformations, they were able to explain why the graph moved in that manner. Analysis of the individual test papers indicated conceptual understanding of the topic. These results also support two existent theoretical views regarding mathematical development; the concepts-first view and the iterative view. Future work should attempt to directly compare this type of instruction to traditional instruction as well as developing similar approaches in other areas of mathematics instruction.

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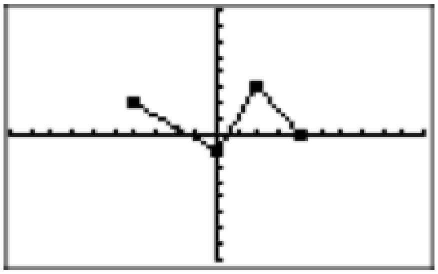
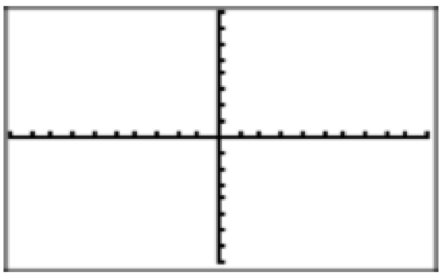
APPENDIX

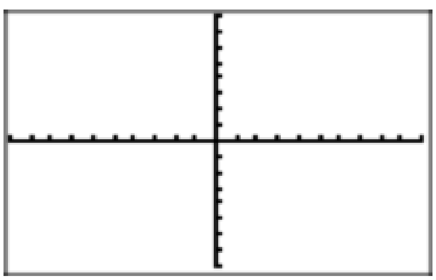
1. The graph of y = f(x) is shown below.



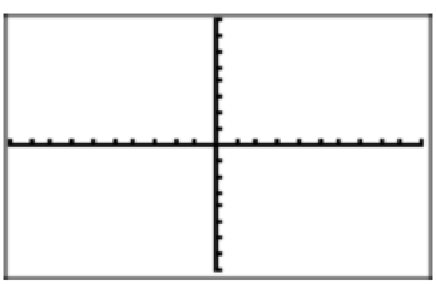
a. On the axes below, sketch the graph of $y = f(x+1)$



b. On the axes below, sketch the graph of $y = f(x)+3$



c. On the axes below, sketch the graph of $y = f(2x)$



2. Describe how the function f(x) moves in order to obtain the graphs of the transformations given below.

a. $y = f(x-2)-1$

b. $y = 3f(x)$

c. $y = f(\frac{1}{2}x)$

3. Complete the table below.

x	-3	-2	-1	0	1	2	3
f(x)	8	-6	7	2	-5	0	9
f(-x)							
-f(x)							
f(x-1)							
f(x)-3							

EGG CARTON FRACTION MODEL

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A major goal of mathematics teacher education is developing the mathematical and pedagogical content knowledge of elementary preservice teachers. Reflective questions on the final exam for a mathematics content course for future teachers were analyzed to identify what the most important mathematics concepts and most helpful activities during the semester. The constant comparative method analysis reported fractions as the most important concept and fraction models as the most helpful activity. Further analysis identified the egg carton fraction model as most helpful, and associated student voices are given.

Teacher educators often review curricular approaches in an attempt to improve student outcomes. This research project examined reflective responses to two questions on the final exam in a content course for preservice teachers. Preservice teachers were asked what the most important concept and helpful activity were for the semester. Fractions were overwhelmingly reported as the important concept and fraction models as the helpful activity.

Many concrete fraction models are available either commercially or by using everyday items. The participants in this research project had the opportunity to explore several fraction models, such as pattern blocks, fraction strips, tangrams, and a model using egg cartons and plastic colored eggs. A second examination of the data identified the egg carton model as the most preferred model. Ashlock (1983) argued that learning should begin with concrete experiences and move through visual representation to abstract thinking. The egg carton model begins with physical eggs in a carton, then moves to visual representation and provides insight into the traditional abstract algorithms for fraction operations. The egg carton model sparked curiosity and encouraged engagement with its bright colors and non-threatening atmosphere. A student was heard to say, “Easter eggs! What are we doing today? I want the purple ones.”

The purpose of this research project was to gain a better understanding of what preservice teachers consider to be important mathematical concepts and helpful activities for mathematical learning and understanding after completion of the first mathematics content course for teachers. Eliciting feedback from preservice teachers through reflective questions on the final exam was one way of considering the effect of curricular approaches.

Literature Review

Preservice teachers' knowledge of fractions is limited (Newton, 2008; Zhou, Peverly, & Xin, 2006); in addition they exhibit low confidence in operating with fractions (Ball, 1990; Newton, 2009). A major goal of mathematics teacher education is developing the mathematical and pedagogical content knowledge of preservice teachers and raising their confidence levels in working with fractions (Tirosh, Fischbein, Graeber, & Wilson, 1998). Working with familiar everyday objects, as opposed to traditional manipulatives designed specifically for mathematics, may engage preservice teachers and raise their confidence more.

Cut-up egg cartons have been used to represent fractional parts of twelve (Hyde & Nelson, 1967). Each color-coded cut-out represented a unit fraction, and pieces were added or subtracted by joining or removing different pieces. However, in this model the "whole" was not physically present during operations; only the numerator was modeled. Some teachers nestled egg carton pieces inside a whole carton to demonstrate equivalent fractions or division of fractions (Monroe & Nelson, 2002; Ott, Snook, & Gibson, 1991). Peck & Jencks (1981) used upside-down egg cartons and string to develop the concept of sharing equal parts with gifted middle school students. May (1992) augmented the model by placing pompoms in the carton to demonstrate addition and subtraction of fractions with like denominators. Pirie and Kieren (1992) began using colored plastic eggs and carton to introduce fraction concepts and model operations. Students then represented the carton of eggs on paper, drawing lines to show equal-sized sections. NCTM Illuminations includes a lesson plan for grades 3-5, using eggs to show fractions as part of a set (Hargrove, 2008). There was no evidence found in the literature of using the egg carton model with preservice teachers.

Methodology

Participants

Elementary and middle school preservice teachers (N = 390) enrolled in the first of three mathematics content courses for future teachers were chosen for this research project. The first course covered number and operation, and was taught by two professors and an adjunct using a common course curriculum. Participants were drawn from their course sections over nine semesters from 2003 to 2010.

Data Collection and Analysis

Two reflective questions were placed at the end of the final exam. Preservice teachers were asked to identify (1) the most important mathematical concept they learned that semester and (2) which class activity was the most helpful. Because responses tended to cross over between questions, the researchers considered both answers when compiling counts.

Researchers chose the constant comparative method (Glaser & Strauss, 1967) to analyze preservice teacher responses. Each researcher coded the data independently using open codes (Maxwell, 2005), then reviewed together to determine a consensus to protect against researcher bias. Emerging themes were categorized and labeled for both questions, and then the highest response categories were revisited for further analysis.

Results

Most Important Concept Learned

Preservice teachers varied in their choice of the most important mathematics concept learned during the semester (see Table 1), with several listing more than one concept. The most frequently mentioned concept was fractions (60%), at a rate five times higher than the next concept.

Table 1.

Most important mathematical concept learned during the semester (may have listed more than one concept).

Most Important Concept	Occurrences (N = 390)	Percent
Fractions	234	60%
Patterns & sequences	46	12%
Other number bases	41	11%
GCF & LCM	41	11%
Prime numbers	20	5%
Operations & properties	18	5%
Factors and multiples	16	4%
Real number system	12	3%
Percents & decimals	11	3%
Base 10 number system	8	2%

The course began with explorations of scenarios based on patterned sequences. Other number bases were explored through group presentations. Number base concepts were practiced with binary operations using base five. Researchers were surprised by the number of preservice teachers listing GCF/LCM as important concepts, and speculated that the use of multiple methods may have been helpful to preservice teachers. The last few weeks of the course covered fraction concepts and operations.

Most Helpful Activity of the Semester

Preservice teachers also varied in their choice of helpful activities during the semester, with several listing more than one. The activity with the highest response rate was fraction models (34%), occurring at almost twice the rate of the next category. Because fractions were the most reported concept and fraction models were the most reported helpful activity, a second round of analysis looked at the subset of fraction model responses. This subset (N = 134) was mined for specific fraction model names (see Table 2). The egg carton model was listed by almost half of the preservice teachers.

Table 2.

Fraction models named as helpful activities during the semester (may have listed more than one model).

Fraction model	Occurrences (N = 134)	Percent
Egg carton model	61	46%
Fraction chart	38	28%
Area model on graph paper	21	16%
Pattern blocks	11	8%
Tangrams	8	6%
Patty paper	6	4%
Fraction cards	4	3%
(Not mentioned by name)	7	5%

A third round of analysis examined preservice teacher comments associated with the egg carton model. Fifty-one reflections had some descriptive commentary related to the egg carton model. Five general categories emerged: visualizing fractions, unique model format, effect on student learning, change in own learning, and associated emotions. These categories and an example comment from a preservice teacher are given in Table 3.

Table 3.

Categorized preservice teacher comments about the egg carton fraction model.

Category	Occurrences (N = 51)	Percent
Helps to visualize or "see" fractions	23	38%
"I learned how much easier it is to work with fractions if you can 'see the picture'."		
Has a unique and engaging format	9	15%
"I also learned that math doesn't have to be hard or boring. A teacher doesn't have to use the same old equipment or worksheets – we can use egg cartons and blocks and so many other things."		
Effect on student learning	9	15%
"I have always had trouble with fractions. I didn't understand why you would multiply with reciprocal fractions to get the dividend. Modeling them on paper or with egg cartons, have helped me out a lot. I understand that teaching modeling before the actual arithmetic will help the student have a better understanding."		
Change in their own learning	8	13%
"I don't remember really having activities to understand fractions as a child. The twelve sections are perfect because can easily split 12 into halves, thirds, and fourths."		
Associated emotions	2	3%
"It also helped trying to color the egg cartons. Made me feel better about fractions."		

Discussion

Sixty percent of the preservice teachers reported fractions as the most important concept learned during the semester; other concepts were mentioned at less than a quarter of that rate. Fraction models were listed as the most helpful activity at twice the rate of other activities. This overwhelming response of fraction concepts and fraction models may be explained by the intense focus during the last few weeks of the semester. However, the final exam covered all topics, concepts and skills for the semester, so they would have also been on the preservice teachers' minds as they worked through the exam. The egg carton fraction model was mentioned more frequently than other fraction models, even though each model was given the same amount of instructional time and homework. Preservice teachers explained that the model helped them visualize fractions and "see" exactly what is happening during operations on fractions. This model is explained below in an attempt to share and disseminate a curricular approach that is self-reported as helpful by preservice teachers.

Egg Carton Fraction Model

This activity introduced fraction concepts in a concrete manner, bridging to a visual representation to support the development of understanding of numerator, denominator, fraction magnitude and operation. The activity included two handouts: a page to develop a fraction glossary and a page to work on fraction operations. Preservice teachers were each given an egg carton full of colored plastic eggs, plastic straws for dividers, and two handouts.

Fraction Glossary Handout

The instructor introduced the egg carton fraction model by presenting a full egg carton and posing questions to elicit the concept of the "whole", represented by 12 eggs placed in 12 equal-sized cups ($\frac{12}{12}$). Then the instructor helped preservice teachers to develop a glossary of fractions with denominators of 12, 6, 4, 3, 2 and 1. Starting with $\frac{1}{12}$, they modeled each unit fraction with eggs, discussed alternative positions and their equivalence, and then colored the representation in on the handout. The instructor indicated the preferred placement of eggs from left to right, similar to reading and the number line. This optimum placement was important with fractions operations later. Subsequent composite fractions were modeled using eggs and then recorded on paper, using color to express the numerator (how many eggs present) and lines to show the equal parts of the denominator (see Figure 1). Identifying unit fractions and then related composite fractions provided an opportunity to think about the magnitude of related fractions. Preservice teachers were then encouraged to use their fraction glossary when learning how to operate with fractions using the egg carton model.

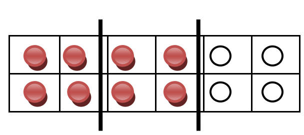

Figure 1: Modeling $\frac{2}{3}$ with color and lines on the glossary handout.

Operating with Fractions

A second handout provided scaffolding for operating with fractions. The instructor modeled an example for each operation. The use of color and lines was emphasized so that individual thinking was more transparent and the teacher could check for understanding. Addition was presented as “joining” of two parts and subtraction as “removing” (crossing out colored circles) part of a part. Rather than “groups” as in whole number multiplication, fraction multiplication was presented as a “part of a part”. Division was presented as a measurement model, where the second fraction was “measured across” the first fraction. Multiplication and division representation on paper involved circling sections of the egg carton as well as marking lines as needed. In both cases the answer was read off of the circled portions. (see Figure 2; handouts and key available by request from authors.)

$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$ $\qquad$ $\frac{5}{6} \times \frac{2}{5} = \frac{1}{3}$ $\qquad$ $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$

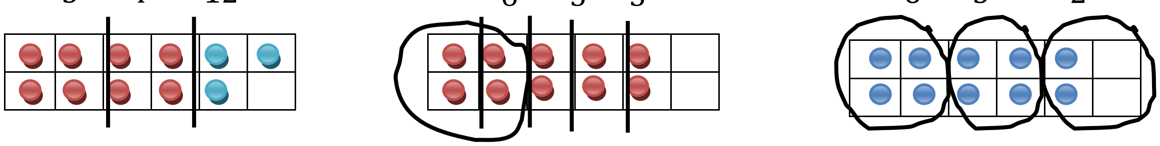

Figure 2: Modeling fraction addition, multiplication and division with the egg carton model.

Conclusion

As has been shown in the literature, preservice elementary teachers have difficulties with fractions. Based on the analysis of reflective responses on final exams, fractions were reported as the most important mathematical topic learned. Specifically, the egg fraction model was found to have the most impact on their learning. By physically manipulating the eggs in the egg cartons, the preservice teachers were building their fractional number sense and gaining a deeper understanding of the meaning of fractions. For example, they could physically see how $\frac{2}{3}$ and $\frac{4}{6}$ represent the same amount – in this model, namely, eight eggs out of a dozen. By modeling fraction operations using concrete and visual representations, the preservice teachers were engaged with the concepts and began to explore what it *means* to work with fractions (add means ‘join’, subtract means ‘remove’, multiply means ‘part of a part’, and divide means ‘how much/many of this part can fit into this part’).

As one preservice teacher wrote, “I have never seen anything like it before and I thought it was pure genius…In all my years of being a student fractions have never stuck with me the way that did. I will defiantly [sic] use them in the years to come.” Another explained, “I have never really understood what fractions were or what I was doing to them when I added, subtracted, multiplied, or divided. After using the egg carton I really understand what is happening.” Clearly, for these preservice teachers, the use of this model was instrumental in developing their deeper understanding of fractions.

The researchers have recently videotaped the egg fraction model lesson with three sections of preservice teachers in an attempt to analyze the approach and delivery of the lesson. Future research could include investigating preservice teachers’ interactions when learning the egg fraction model as well as the pedagogical aspects of the egg fraction model they identify. Further analysis of the nature of their comments with regards to this model is needed.

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A MATHEMATICS DIFFERENTIATION MODEL TO HELP NEW TEACHERS ENGAGE ALL STUDENTS

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This study tests a differentiation model that is designed help new mathematics teachers write lesson plans that address the needs of all classroom students. The differentiation method was developed by collaborating with mathematics and science classroom teachers and university faculty. The theoretical framework came from the Response to Intervention system that identifies and classifies students' academic progress. In-service teachers reviewed the format and identified a key missing element. A mathematics student teacher pilot tested the method and gathered data. The result of this collaborative process created a four-tiered format for lesson differentiation to engage all students in learning mathematics.

In order to thrust the economy of the United States (U.S.) into active participation in a global economy, all U.S. students need skills as critical thinkers and problem solvers. Educators must provide the opportunity for learners to engage in the educational process to achieve this goal. All students should be included in the educational process based on the democratic philosophy and propositions stated in the *Constitution* of the United States (Business Roundtable, 2005; Committee on Prospering in the Global Economy, 2007; Gutmann, 1987). American education should provide opportunities for all students to engage in the learning process.

While new mathematics teachers desire a "one size fits all" format for their lesson planning, it is imperative that they understand that within each classroom, whether the classes are content specific (such as a calculus class), tracked, or homogeneous, there is an array of different learning happening within any given classroom. To meet the needs of every student, teachers need to know their students to make appropriate changes to lessons that meet each student at a level that enables that student to actively engage with and learn the mathematics concepts.

Master teachers differentiate their lessons to meet the needs of all students in a classroom. They observe a student struggling with a mathematics concept and respond with variations or modifications to the lesson requirements, adjust the challenges, alternate the implementation, or provide additional information in the middle of a class. These changes keep students on track and learning. How many years does it take to master this skill? That would depend upon the teacher, the diversity of the student population, the district curriculum demands, and the academic freedom allowed. Many experienced teachers are learning through professional development opportunities how to differentiate using multiple modalities: intellectual, gender, socio-economic status, location (urban, suburban, rural), or by learning or religious preferences. While there have been a multitude of demands for teachers to differentiate their teaching, there are few procedures that outline how to differentiate lessons. The question examined in this study is: does a Four Tier differentiation model assist new mathematics teachers to engage all students in mathematics learning?

The primary responsibility of professional educators is to address the needs of all students in a classroom. The National Board for Professional Teaching Standards (NBPTS) identifies as its Core Proposition #1 that teachers should know their students' individual differences and accommodate for those differences in their practice (NBPTS, 2008). PRAXIS pedagogical evaluation states that teachers should know their students in the first criteria of Domain A - Planning for Teaching (Danielson, 2007). Teachers can select appropriate learning techniques, methods, and environments that foster students' active engagement in the content. Being informed about the background knowledge, experiences, and learning preferences of students enables mathematics teachers to select learning techniques, strategies, and environments that engage students in active content learning.

The educational community responded to the need to address diverse learning skills of all students. Tomlinson (1999) noted that teachers should discover multiple student interests and use learning modalities as avenues to engage students. She recommended that varying the rates and degrees of instruction complexity keep all students engaged with the content. NCTM's *Current Collection of Tips (*2009) on differentiated learning noted that teachers should "Focus on the differences that exist, value the diversity, and allow each student the opportunity to shine. Instruction may be differentiated in content, process, or product according to the students' readiness, interests, or learning style."

First year teachers are expected to be able to add value to their students' knowledge base from the moment that they are hired. Time for teachers to develop their pedagogical content knowledge is dramatically reduced with the requirements of value adding assessment methods used to evaluate teacher effectiveness. Teaching skills that developed over several induction years now must be part of a newly hired teacher's portfolio of skills. This study presents one approach that helps new mathematics teachers differentiate lessons in order to assist students' entry to learning mathematical content standards and increases student achievement.

Tomlinson and McTighe (2006) combined cognitive developments with teaching strategies to create an action plan for educators that included differentiation in multiple modes. They prescribed how lesson differentiation could be accomplished when designing the content for understanding. The authors identified this method as Responsive Teaching where teachers made modifications to a lesson that helped students to access major ideas and skills, thus enabling students to make sense of concepts that are big ideas about specific content.

Response to Intervention (RtI) (National Center for Learning Disabilities, n.d.) is a method of differentiation used by Intervention Specialists to coordinate assessment practices and instructional changes in pedagogy to keep students from slipping through the educational system into behavioral issues or educational deficits. This methodology responds to the No Child Left Behind Act mandate to help all children achieve state requirements (NCLB, 2001; National Center for Learning Disabilities, n.d.). The RtI model begins with the notion that the basic lesson reaches 85% of the classroom students. A second group of students constitutes the next 10% who need additional support of some kind to grasp the lesson. Dramatic intervention and the expertise of an intervention specialist would be needed by the last 5% of the class. This group requires instruction tailored to the needs of the individual student in order to achieve the lesson objective. RtI requires that teachers plan for multiple levels of entry to the content.

Collaboration

It became clear with the requirements of the No Child Left Behind Act of 2001 that more needed to be done to help all teachers address the varying intellectual needs of their students. Working with a group of high school teachers who asked for a book study group on differentiation, we used Tomlinson and McTighe's *Differentiating Instruction through Understanding by Design* (2006). The group concluded that differentiation at the high school level was done by the achievement levels of courses, but the teachers needed to make additional adjustments to lesson implementation within each classroom in order to engage all students. However, the teachers were not able to identify one, specific differentiation format that new teachers could replicate when planning.

Differentiation based on students' prior knowledge levels was the next area that I examined. Collaborating with a university colleague, we researched and wrote an article

(Driskell & Author, 2007) that identified tiers of student understanding based on the van Hiele levels of geometric understanding to help fourth graders enter the examination of shape at his or her developmentally appropriate level.

In a summer institute for professional development, I required in-service teachers to write lesson using the RtI three tiers. In addition to differentiating a lesson by finding multiple entry points, the teachers identified how they would assess each level. The teachers quickly pointed out that the format left out one group of students – students who achieved at a very high level – the accelerated students.

Using the information collected over several years of research and collaboration, I formatted a four tier method for new mathematics teachers to differentiate lessons. The format borrowed heavily from the RtI approach but addressed a wider array of students and had one major difference, no student would be locked into a tier. Assessment and review of the mathematical work of the student needed to be done each day. This format used the three tiers from the RtI model and added a tier for accelerated students resulting in a four level differentiation model. This differentiation model included the following tiers to address the needs of all students in a mathematics classroom:

Tier 0 – the Accelerated students. These students grasp the mathematical concept taught with ease. This tier must meet the needs of students who absorb a mathematics concept with such speed that these students complete their requirements soon after the work is assigned. Additional challenges, not busy work, but intellectually challenging mathematics should be assigned to these students. This level would address approximately 2-3% of the class.

Tier 1 – the Majority of the students. This is the grade appropriate group of the mathematics lesson. This level correlates to state curriculum requirements. The planning of the mathematics material for this level represents how a new mathematics teacher would conceive and present the original mathematical concept before making variations for Tiers 0, 2, and 3. This tier addresses between 80-85% of the class.

Tier 2 – Those students rooted in concrete learning, or need to use manipulatives to construct a mathematical understanding of a concept should be addressed in this tier. This level helps the new mathematics teacher prepare specific materials or additional activities for students who do not grasp the mathematical concept of the lesson at the first presentation. These alternatives need to be presented within the timeframe designated for the lesson. This planning is for approximately 10 -15% of the class.

Tier 3 – Those students whose needs are not met in Tier 2 and may need more time to master a mathematical concept, or experience more interactions with the teacher, or need more mathematical examples at the concrete level to grasp the concept reside in this tier. Many of these students have individual educational plans (IEPs). The new mathematics teacher needs to identify what difficulties these students have when learning mathematics. All plans for these students must remain within the parameters of the IEP. This tier represents 3-5% of the class.

One major difference between the RtI and the Four Tier approach to differentiation is the fluidity of the Four Tier approach system. Teachers can move students from one tier to another on a daily basis. Formative assessment methods help determine what tier a student occupies on a given day. Tier placement is not a permanent identification, nor does the new mathematics teacher need meetings with other faculty to move a student into another tier. Students can change daily as the new mathematics teacher employs good assessment strategies that identify the needs and achievements of each student.

Methodology

This study of tiering mathematics lessons used a comparison of tests to determine if there was a difference between using a teaching approach that addressed all students with the same teaching mode or differentiating the mathematics material. A comparison of test scores was done to measure student achievement using each method.

Participants

The participants in this study included a preservice teacher and a mathematics class of 22 high school students. The male pre-service teacher was earning licensure in Adolescence to Young Adult Mathematics (grades 7-12) in the student teaching semester (final term) of his four year university experience. The 22 high school students were from a lower-middle income school district in a Mid-Western state. The pupil population of the school district is composed of 83% Caucasian, 10% African American, 3% Multiracial, 2% Hispanic, and 2% Asian. The percentage of the population that is economically disadvantaged is 25%. The pupil attendance rate is 93%. The number of pupils per teacher averages at 19 pupils per teacher. The district spends $8,635 per pupil annually (Great Schools, Inc,. 2010).

Procedure

The pre-service teacher planned two units of content in a pre-calculus class: one unit taught, reviewed, and tested in the presentation mode used by the cooperating teacher, and a second unit taught in the same presentation mode with a differentiated review prior to the test. He identified each student on the data report as a number to protect the anonymity of the students and for purposes of matching the first test with the second test. By using prior assessments, the mathematics pre-service teacher determined that his students fit into three tiers: Tiers 0, 1, and 2. This was a concern of his at first, but justified by test scores. He noted that there were no students with an individual educational plan, nor were any students identified as needing the additional planning required of Tier 3.

Using an inquiry-based lesson on the law of sins for one week, he conducted a review of the content, and then tested the students producing the first data set. He taught a second lesson using an inquiry-based approach on the law of cosines. During the review portion of the unit, he created three review activities that addressed the needs of Tiers 0 though 2. The Tier 0 students received worksheets with review questions that were abstract in nature and challenged the students to solve difficult problems about the laws of cosines. The Tier 1 students were given problems similar to those in the textbook. The Tier 2 students were presented work that had addressed sensory learners and were more visual representations of cosines. The pre-service teacher had these students cut out shapes that would be used to model specific law of cosine examples. While these students manipulated the shapes, they were focused on the link between the shapes constructed and the law of cosines. The next day, all students were administered the same test on the law of cosines producing the second data set.

Data Collection

Data to evaluate the effect of the four tier format for differentiation were collected from the tests administered as summative assessments. Identification of students into the tiers was done prior to the lesson presentations. The first data set came after the students in the pre-calculus class took a test concentrating on a unit taught using the cooperating teacher's lesson presentation model. The second data set was the grades from a test given to the students after a unit in which a differentiated review was provided to the students prior to the test.

Results

The data results report the number of students in each tier and the analysis of the scores of the two tests conducted in this study. The data were examined as individual measures, aggregated measures, and the significance of increases or decreases in student achievement.

Using grades earned on class work done prior to this study, the preservice teacher placed the 22 students into only three tiers. The tier populations was an almost even distribution of students between tiers 0, 1, and 2 with six students in tier 0, eight students in tier 1, and eight students in tier 2.

The first and second test scores were used to compare the increase or decrease of points earned by the students. The students' earned scores that fluctuated by several points of increase or decrease. Sixteen students increased their scores while six students achieved lower scores when the differentiated review was implemented. (See Figure 1).

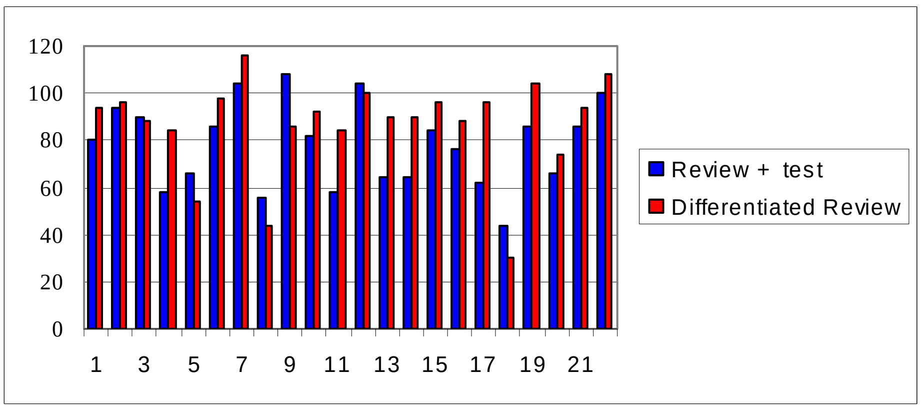

Figure 1. Comparison of test 1 and test 2 student scores.

When aggregating the test score results by tier, the results identified that all tiers increased scores after the differentiated review was implemented. The aggregated totals increased by tier with the scores of Tier 0 increasing from the first test by 30 points. Tier 1 aggregated scores increased from test one to test two by 64 points. Tier 2 aggregated scores from test one to test two increased by 114 points. To determine the average increase in scores, the aggregated increases were divided by the number of students in each tier with the following results: Tier 0 scores increased by 5 points each, Tier 1 scores increased by 8 points each, and Tier 2 scores increased by 14.25 points each. Of the students whose scores increased when a differentiated review was used, 16 students representing 73% of the class improved their performance on the second test.

There were students who received lower scores on the second test when the differentiated review took place. Six students' scores decreased, representing 27% of the class when a differentiated lesson review was used. The largest drop came in Tier 0 where one student lost 22 points from the first test to the second test. Three students in Tier 1 dropped scores of 2, 4, and 8 points each. Two students from Tier 2 dropped their scores by 12 points each. Increases in the test scores varied by tier. Tier 0 individual test scores increased from 2 points to as much as an increase of 18 points. The individual test scores of Tier 1students increased from 2 points to a high increase of 26 points. The greatest increases in individual scores were found in Tier 2 scores where the smallest increase was 10 points to the highest individual test score increase of 34 points. One student had a 34 point score increase in his/her score while two other students within Tier 2 improved their test performances by 26 points each.

The results of the paired t-tests found that for the whole class, the grades were significantly different when this four tier method of instruction was used. The two-tailed $p \leq 0.0135$. The mean of test one (normal instruction) minus test two (differentiated instruction) equaled -8.55 at a 95% confidence interval of the differences between -15.13 to -1.96. The t value equaled 2.6984 with 21 degrees of freedom and a standard error of difference of 3.167. The summary of the details for this data group are in Figure 2.

Group	Test Normal Instruction	Test-Differentiated Instruction
Mean	78.09	86.64
SD	17.95	20.26
SEM	3.83	4.32
N	22	22

Figure 2. Whole Class Paired t-test results.

The results of the paired t-tests done on Tier 0 test scores found that the differences were not significantly different with a $p \leq 0.4291$. The mean of test one (normal instruction) minus test two (differentiated instruction) equaled -5.00 at a 95% confidence interval of the differences between -19.94 to 9.94. The t value equaled 0.8600 with five degrees of freedom and a standard error of difference of 5.814. The Tier 0 summary details are in Figure 3.

Group	Test Normal Instruction	Test-Differentiated Instruction
Mean	90.67	95.6
SD	9.18	5.85
SEM	3.75	2.39
N	6	6

Figure 3. Tier 0 Paired t-test results.

The results of the paired t-tests done on Tier 1 test scores found that the differences were significantly different with a P value of 0.0276. The mean of test one (normal instruction) minus test two (differentiated instruction) equaled -9.25 at a 95% confidence interval of the differences between -17.14 to -1.36. The t value equaled 0.2.7722 with 7 degrees of freedom and a standard error of difference of 3.337. The summary of the details for the Tier 1 data are in Figure 4.

Group	Test Normal Instruction	Test-Differentiated Instruction
Mean	85.50	94.75
SD	16.38	13.09
SEM	5.79	4.63
N	8	8

Figure 4. Tier 1 Paired t-test results.

The results of the paired t-tests done on Tier 2 test scores found that the differences were not quite significantly different with a P value of 0.0573. The mean of test one (normal instruction) minus test two (differentiated instruction) equaled -14.25 at a 95% confidence interval of the differences between -29.08 to 0.58. The t value equaled 0.2.2717 with 7 degrees of freedom and a standard error of difference of 6.273. The summary of the details for the Tier 1 data are in Figure 5.

Group	Test Normal Instruction	Test-Differentiated Instruction
Mean	61.25	75.50
SD	10.74	19.79
SEM	3.80	7.00
N	8	8

Figure 5. Tier 2 Paired t-test results.

Discussion

The results of this study provide evidence that a new mathematics teacher can increase high school student mathematical achievement when they differentiate their lessons using this Four Tier format. This study focuses on using a specific differentiation by tiers approach. The study occurs in one pre-calculus classroom of 22 students prepared by a pre-service teacher. The resultant data show an increase in student achievement for 73% of the class when compared to the test scores from a content unit taught by the same mathematics preservice teacher when he used a uniform presentation format for all students. The tier with the scores that increased the most was Tier 2 where the range of scores increased from 10 points to a maximum of 34 points when the scores were compared with the first test that used one instructional method for all students. Bringing the Tier 2 students into evaluation ranges that were much higher than their normal testing achievement levels was a point of great pride for the mathematics pre-service teacher.

With the encouragement of the increased student scores, a caution must be noted regarding the number of decreased student scores. Lower scores were achieved by approximately 23% of the students on the second test. While this was only one experiment with differentiation, the results impacted the mathematics preservice teacher's notions of how content should be presented to students in order to engage all students.

Issues Implementing Lesson Differentiation with New Mathematics Teachers

New mathematics teachers needed to comprehend that tiered lesson modifications must be accomplished within the timeframe of the lesson being taught. This point was the crux of the very difficult issue for mathematics preservice teachers when trying to implement differentiated lessons. While this four tier differentiation lesson model does not prepare new mathematics teachers for all learners, the method does start them examining how students think and learn mathematics.

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WRITE IS RIGHT: USING GRAPHIC ORGANIZERS TO IMPROVE MATHEMATICAL PROBLEM SOLVING

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Students use graphic organizers successfully in the writing process. This paper describes graphic organizers and their potential mathematics benefits for both students and teachers; describes the specific graphic organizer adaptations we did for mathematical problem solving; and discusses some results using the four-corners-and-a-diamond graphic organizer with 186 inner-city, minority middle school students.

Improving student problem-solving skills and abilities is a major, if not *the major*, goal of school mathematics (National Council of Teachers of Mathematics, 1989; 1995; 2000). This paper describes a novel approach to mathematical problem solving derived from research on reading and writing pedagogy, specifically, research indicating that students use graphic organizers to organize their ideas and improve their comprehension and communication skills (Goeden, 2002; National Reading Panel, 2000).

This research used classroom action-research methodology on a problem-solving instruction. Purposefully, we utilized graphic organizers to increase middle school student achievement in five areas of the state mathematics assessment in open-response problems (Zollman, 2009a; Zollman, 2009b). The following describes graphic organizers and their potential mathematical benefits for both students and teachers, elucidates the specific graphic organizer adaptations we used for mathematical problem solving, and discusses some of the research results from using the *four-corners-and-a-diamond* mathematics graphic organizer.

Background and Benefits of Graphic Organizers

A graphic organizer is an instructional tool to assist students in organizing and structuring information and concepts. It promotes the use of relationships between concepts. Furthermore, the spatial arrangement of a graphic organizer allows the student and the teacher to identify missing information or absent connections in their strategic thinking (Ellis, 2004).

Middle school teachers already use many different types of graphic organizers in the writing process. All share the common trait of making the process of thinking into a *pictorial (graphic) format*. This reduces, and helps organize, information, concepts, and relationships. The student completes this *graphic format*. The learner does not have to process as much specific, semantic information to understand the information or problem (Ellis, 2004). Graphic organizers allow (and even expect) the student to sort information as essential or non-essential; structure information and concepts; identify relationships between concepts; and organize communication about an issue or problem. Prior research found students use graphic organizers to organize their ideas and improve their comprehension and communication skills (Goeden, 2002; National Reading Panel, 2000).

Initial thinking is not a linear activity, especially in mathematical problem solving. Yet, the result of problem solving – the written solution – looks like a linear, step-by-step procedure. Good problem solvers have different thoughts when first presented with a problem. These random, brainstorming ideas may or may not be useful. Using a graphic organizer allows random information and ideas to be recorded but not processed. A student can later reflect upon usefulness of the information and ideas. If the information and ideas help the student make relationships between concepts, then it is essential. Using a graphic organizer allows a student quickly to organize, analyze, and synthesize one's knowledge, concepts, relationships, strategy, and communication. It also gives every student a starting point of the problem-solving process (Zollman, 2009a; 2009b).

Adapting a Graphic Organizer for Mathematical Problem Solving

Figure 1 depicts the *four-corners-and-a-diamond* mathematics graphic organizer. This graphic organizer is modeled after a *four squares* writing graphic organizer described by Gould and Gould (1999). Our *four-corners-and-a-diamond* mathematics graphic organizer has five areas:

i. What do you need to find?
ii. What do you already know?
iii. Brainstorm possible ways to solve this problem.
iv. Try your ways here.
v. What things do you need to include in your response? What mathematics did you learn by working this problem?

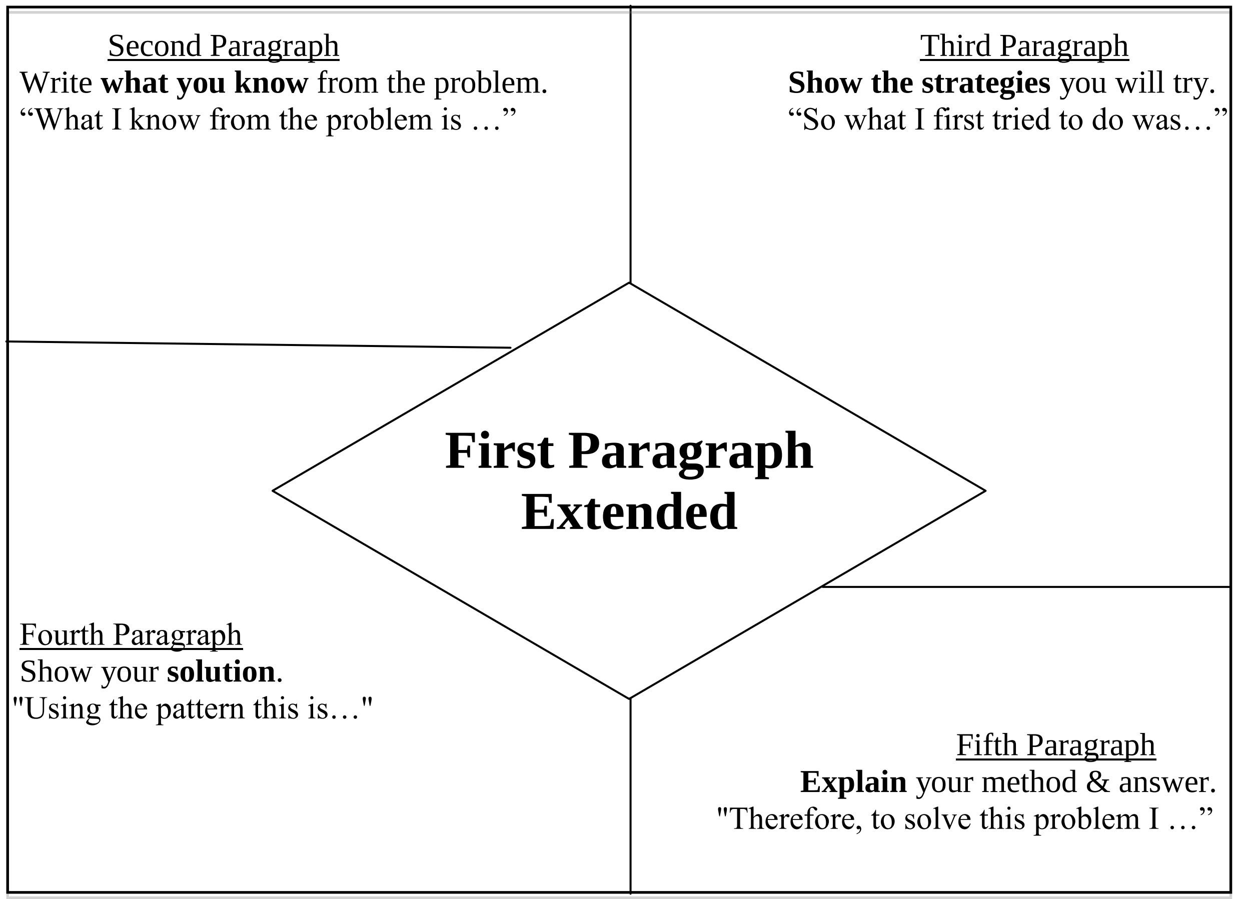

Figure 1. Four-Corners-and-a-Diamond Mathematics Graphics Organizer

So how does the use of *four-corners-and-a-diamond* graphic organizer differ from the traditional Polya’s four-step mathematical problem-solving hierarchy? In terms of objective, it does not. Obviously, the *four-corners-and-a-diamond* graphic organizer wants students to understand the problem; devise a plan; carry out the plan; and look back (Polya, 1944). However, by having the non-linear layout of the graphic organizer, the student is not expected to do these “steps” in a hierarchical, procedural order that some students misapply. It is the implementation process, i.e., how students do their response, which is the important aspect of the *four-corners-and-a-diamond* graphic organizer (Zollman, 2009a; 2009b).

The pictorial orientation allows students to put down their ideas in *whatever* order they occur. If students first think of the unit for their final answer, then this is recorded in the fifth, bottom-right area. This idea (the unit) then is not needed in the short-term memory, as a reminder is recorded. If students first think of a possible procedure for their answer, then this is recorded in the third, upper-right area. The *four-corners-and-a-diamond* graphic organizer allows, even encourages, students to not feel they must do their problem solving strategies in a hierarchical order. One can work in one area and then later work a different area. It also shows that completing a problem-solving response has several related, but different aspects.

A written response is not begun until some information or concept is in all five areas. The *four-corners-and-a-diamond* graphic organizer especially encourages students to begin working on a problem before they have an identified solution method. As in the four square writing method, the students then organize and edit their thoughts by writing their solution in the traditional linear response, using connecting phrases and adding details and relationships. For the open response write up, students first state the problem, then the given information, next their methods for solving the problem, after that their mathematical work procedures, and finally their final answer and conclusions.

The graphic portion of the organizer allows all students to fill in parts of the solution process. It bolsters students to persevere – to "muck around" working on a problem. Further, teachers quickly can identify where students are confused in solving a problem.

The teacher can model and have students work in groups when introducing the *four-corners-and-a-diamond* graphic organizer. When working in groups, students always are amazed that many problems can be worked in more than one way, and that different people start in different places when solving a problem. In their small group discussions, students identify relationships between the areas in the graphic organizer and among the various solutions.

A possible secondary benefit is seen on student scores on the open-response math problem-solving items of state mathematics assessment. In most states, there is a scoring rubric. In Illinois, the scoring rubric has three categories, namely Mathematical Knowledge, Strategic Knowledge, and Explanation (Illinois State Board of Education, 2005). Each category is individually scored from 0 to 4 points (from no attempt, to limited, to some, to most, to complete). Traditionally, low-ability students do not even show any work in one of more categories in their response. Average-ability students have disorganized responses. In addition, higher-ability students skip steps in their explanations. The *four-corners-and-a-diamond* graphic organizer helps each type of student do a more complete response in each of the three categories, and thus, receive a higher score.

Methodology

This research was part of a math-science partnership project in an inner city school district. The district's three middle schools (grades 6-8) have a history of poor mathematics achievement on the state assessment. In the previous year, 65% of 8th grade students did not meet expected achievement scores on the state mathematics assessment.

Nine middle school mathematics teachers were part of the project to increase student achievement by: increasing teachers content expertise in five critical state learning standards, (algebra, geometry, probability, statistics, and measurement (Illinois State Board of Education, 1997); increasing pedagogical teaching skills; and increasing teacher understanding and application of educational research to enhance classroom practice. For the goal of understanding and applying educational research, the nine middle school teachers decided to use the open-response mathematics questions of the previous state assessment as the focus of their action research. The teachers used the *four-corners-and-a-diamond* graphic organizer weekly on open-response items with 186 of their students in the instruction of these five critical areas. The teachers measured pre- and post-test scores of their students using the state's 4-Point Scoring Rubric (Illinois State Board of Education, 2005), to see the impact on the five mathematics areas during the school year.

Results Using Graphic Organizers

All nine teachers' action research projects reported dramatic improvements in students' mathematics scores on open-response items by implementing the *four-corners-and-a-diamond* graphic organizer. The state 4-Point Scoring Rubric for open-response items gives scores on Math Knowledge, Strategic Knowledge, Explanation, and Overall Extended Response (Illinois State Board of Education, 2005). On the pretest items, only 4% in Math Knowledge, 19% in Strategic Knowledge, and 8% in Explanation of the 186 students were scored at the "meets" or "exceeds" levels on the open-response items. After instructing students in using the graphic organizer via mathematical problem solving, 75% in Math Knowledge, 68% in Strategic Knowledge, and 68% in Explanation of the 186 students scored at the "meets" or "exceeds" levels on the post-test items (Zollman, 2009b). Tables 1-4 show the mean score of the 186 students on the open-response items.

Table 1

Mathematical Knowledge Score*

	n	*Mean*	*SD*	*Z-score*	*p-value*	*Effect Size*
pretest	186	0.83	0.8526	–19.8849	0.000	1.94
posttest	183	2.93	1.3303			

**range 0 to 4*

Table 2

Strategic Knowledge Score*

	n	*Mean*	*SD*	*Z-score*	*p-value*	*Effect Size*
pretest	186	1.52	0.9312	–11.6049	0.000	1.16
posttest	183	2.79	1.2755			

**range 0 to 4*

Table 3

Explanation Score*

	n	*Mean*	*SD*	*Z-score*	*p-value*	*Effect Size*
pretest	186	0.83	1.0393	–15.3907	0.000	1.51
posttest	183	2.67	1.4187			

**range 0 to 4*

Table 4

Overall Extended Response Score*

	n	*Mean*	*SD*	*Z-score*	*p-value*	*Effect Size*
pretest	186	1.06	0.9972	–15.4067	0.000	1.50
posttest	183	2.80	1.345			

**range 0 to 4*

These data were self-collected and self-scored (using the state Scoring Rubric) by each teacher. The Overall Extended Response student scores rose from a 27% (1.06/4.00) average on the pre-test to 70% (2.80/4.00) average on the post-test. The use of the graphic organizer in mathematical problem-solving tasks greatly aided the students to coordinate their mathematical ideas, methods, thinking and writing. The graphic organizer helped students coordinate various parts of mathematical problem solving: a) what is the question, b) what information is known, c) what strategies might be used, d) how to do the operations, procedures, algorithms of the strategy, e) what explanations and reflections is needed to communicate the method(s) of solution (Zollman, 2009b).

In particular, the teachers' action research with their students found the use of graphic organizers in mathematical problem solving to be very efficient and effective for all levels of students. The teachers saw their lower-ability students, that normally would not attempt problems, now had partial solutions written. For average-ability students, the organizer helped to organize thinking strategies. For high-ability students, the organizer improved their problem-solving communication skills (Zollman, 2009b). Students now have an efficient and familiar method of writing and communicating their thinking in a logical argument.

Summary

Expanding and improving students' mathematical knowledge to help them problem solve is one of the highest priorities for teaching middle school mathematics. This instructional approach not only assists in content knowledge and in strategic knowledge, but also improves mathematical communication skills. This research found that the proper use of the mathematics graphic organizer *four-corners-and-a-diamond* to be an extremely useful instructional method in the middle school mathematics classroom.

Students should improve their problem solving abilities with any instructional intervention. Many effects can positively influence learning, e.g., the curriculum, the student, the class and the teacher. However, the graphic organizer initiated (from the teachers' viewpoint) many of the beneficial influences in student problem solving (Zollman, 2009b).

The crucial factor in all instructional methods is how it is used. If *four-corners-and-a-diamond* graphic organizer is used as a linear, systematic procedure to teach problem solving, it will succeed sporadically. In fact, all teaching *about* problem solving has intermittent achievement (Lester, 1985). Giving students a chart of Polya's four steps (1944) in problem solving or a graphic organizer sheet may assist a student's learning of the steps of problem solving. However, students often are bewildered about where to start a problem, confused by essential vs. non-essential information, or forget to communicate important steps and reflections in their solutions.

Allowing students to first use their own thinking (then reflect, revise, and re-organize their knowledge, strategies, and communication) assists them to learn to improve their problem solving abilities. Initially teaching about problem solving in a hierarchy of procedural steps is neither efficient nor effective. Our results coincide with other problem-solving findings; teaching *via* problem solving is the key instructional process (Lester, Masingila, Mau, Lambdin, dos Santon, and Raymond, 1994).

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METAPHORS FOR MATHEMATICAL PROBLEM SOLVING

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Current research in mathematical problem solving suggests that language, and specifically metaphors, are influential in a student's ability to perceive, solve, and learn from a mathematics problem. This paper summarizes the results of a pilot study in which high school students attempted to solve and justify three mathematics problems. The student was video recorded and then allowed to watch himself/herself solving the problems. The student was asked to comment on the process used while solving the problems. The analysis and results focused on the student's use of conceptual metaphors and their influence on the student's performance.

Metaphors are a means to relate experiences through language, thought, and action. The relationship between the experiences of the teacher and the student are vital to mathematics education. Specifically, teachers and students share an experiential set: solving mathematics problems. However, the student's and teacher's perspective of what constitutes mathematical problems and/or solutions are complex in structure. (Erlwanger, 2004; Lakatos, 1999; 1976; Lesh & Zawojewski, 2007; Pólya, 1945; Schoenfeld, 1992). Metaphors are culturally designed to make these implicit, differing perspectives explicit. Moreover, they have been found to encourage and incite cognition (Lakoff & Núñez, 2000; Sfard, 1997) .

This paper summarizes the results of a pilot study designed to enquire how metaphors are used in mathematical problem solving. Specifically,

Q1. How do metaphors help in understanding a problem?

Q2. How do metaphors guide the process of solving a problem?

Q3. What metaphors are used to describe mathematical problem solving?

Q4. How do metaphors connect problem-solving and proof?

Q5. How can knowledge of conceptual metaphors help teachers improve student learning?

Theoretical Framework

The theoretical framework for this study evolved from the firm belief that mathematics is embodied (Lakoff & Núñez, 2000). Concomitantly, this research perceives of mathematics education under the philosophical axiom of cognitive science; that is, one can understand and interpret how people learn (Gardner, 1987). Within this realm, this study focuses on the influence of metaphors to one's ability to learn from and solve mathematical problems.

Polya (1945) emphasized two types of problems in mathematics: problems to find, and problems to prove. Within both types of problems, Polya (1954) recognized the significance of analogies. Analogous problems can be identified and used to solve foreign mathematics problems. This is the springboard from which this article begins because it was through this branch of thought that Polya and others began to see the influence of language in solving problems.

Polya's linguistic inspiration was paralleled in the 1970's by cognitive scientists, whose emerging field was a culmination of artificial intelligence, cybernetics, anthropology, psychology, philosophy, and linguistics (Gardner, 1987), and now education. This interdisciplinarity attracted mathematics educators such as Alan Schoenfeld (1985) to model students' approaches to problem solving. Schoenfeld's attempts to interpret how students solve mathematical problems demonstrated a high level of complexity which suggested a categorization of problem-solving characteristics rather than a sequential how-to model. Schoenfeld's initial categories included heuristics (as reinvented by Polya, 1945), resources, controls, and beliefs. He found that student's expression of their problem-solving process emphasized aspects of control, beliefs, and explicit knowledge of their own cognition. He clarified this as metacognition, a valuable skill students possess (Schoenfeld, 1992). Combining Schoenfeld's concept of metacognition with Polya's concept of analogy suggests that the language through which students express their cognitive process may demonstrate how students discern between isomorphic (analogous) and non-isomorphic mathematics problems.

The language that the students use to express metacognition is insulated by experience. In this manner, mathematics is embodied (Lakoff & Núñez, 2000). One's knowledge of mathematics is dependent upon one's perspective and experiential learning of that knowledge. For example, Lakoff (2000) demonstrates how the understanding of limit is complex and embedded within one of two conceptual metaphors. One can think of limits graphically; claiming that as *x approaches* a number, the function will also *approach* a number. This metaphor views limits as motion of a small object along the curve, hence the need to "approach". However, the mathematician Weierstrass reconceptualized the notion of limit by introducing the proximity metaphor through which epsilon-delta language has developed in analysis. Weierstass viewed limits by saying; if x is within *the proximity* of a number, then the function will also be within the *proximity* of a number. Both metaphorical perspectives are valuable, and distinct in their techniques of proof. However, the logic necessary to solve limit-based problems is analogous in both metaphors. Thus an elusive bond exists between problem solving and proofs in which metaphors are squarely centered. This is the foundation for Q4.

Polya's concept of analogy is distant from metaphors in linguistics, but surprisingly close in cognition. Analogies reference two concepts already firmly defined in the learners' mind for purposes of connecting their meaning (Sfard, 1997). However, metaphors frequently model a new conceptual structure with a pre-existing structure. The accommodation of known structures into new concepts can define the new concept, and is considered an aspect of conceptualization (Kövecses & Benczes, 2010; Sfard, 1997). Justifiably, Sfard (1997) refers to such conceptual metaphors as implicit analogies. Hence for mathematical problem solving, application of analogies follows the learner's understanding of conceptual metaphors.

Linguists classify these conceptual metaphors into three hierarchal categories: structural, ontological, and orientational (Kövecses & Benczes, 2010; Lakoff & Johnson, 2003; 1980). In all three conceptual metaphors, there is a source domain and a target domain. The source domain is the experientially-known domain and the related concept is the target domain. Thus in the metaphorical linguistic expression "The solution escapes me", the target domain is solutions while the source domain is prey. Hence the conceptual metaphor would be read as "SOLUTIONS ARE PREY". It is important to note that despite the use of the being verb "are", the phrase is unidirectional (Target→Source). Structural metaphors strive to describe a complex concept, such as time, in terms of a concrete experiential object, such as a limited resource, i.e. "Don't *waste* my time". Ontological metaphors provide target domains with less structure and a new reality in which they may be defined. Personification is regularly ontological, as is the phrase "the solution escapes me". Orientational metaphors are the most difficult to relate to experientially according to linguists. They are a broad concept with a specific direction inherent in our development as humans. The metaphorical linguistic expressions "Things are looking up" and "He fell ill" are examples of the conceptual metaphor "HEALTHY IS UP". How these metaphors directly influence a student solving mathematics problems is the focal point of Q1, Q2, and Q3. Applying this metaphorical influence pedagogically is the purpose of Q5.

Research Design

This pilot study used a naturalistic paradigm (Donmoyer, 2001) to study how metaphors influenced student's problem solving because the lack of current research within mathematics education (Sfard, 1997) mandated trustworthiness. Phenomenological inquiry (Short, 1991) was used to search for the essence of what students deemed as mathematical problem solving so as to limit the assumptions by the researcher. Thus, students were chosen according to a list of criteria that indicated the student had a propensity towards mathematics and expressing their thoughts.

Nine students at a suburban high school in Ohio volunteered for the study: 3 freshman, 2 sophomores, and 4 juniors. Each student met with the researcher individually after school for an hour. The students were given the three mathematical problems shown below:

P1. Imagine you had a piece of string. How would you bend this string to make a triangle bounded by the string with the greatest area?

P2. Humans have classified numbers on the number line into two categories, rational and irrational. Rational numbers are those that can be written as fractions, irrational numbers cannot be written as a fraction. Suppose I have an irrational number. If I add one to that number will it be rational or irrational?

P3. How could you cut a cylindrical birthday cake so that you have 8 slices using only 3 straight cuts with a knife?

The techniques and justification for each problem varied mathematically to identify differences or similarities in problem-solving techniques and metaphorical conceptualization. Moreover, the problems were specifically designed to be metaphorically sterile so as to evoke conceptual metaphors from the students without bias. The problems could be done in any order and manipulatives, including cork board, dry erase markers/board, string, pencils, paper, calculators, thumb tacks, pipe cleaners, and straight edges, were available to help the students.

As Steffe (1983) poignantly noted in studies involving children solving problems, there are multiple interpretive mediums involved. The experience of the student is expressed by the student, interpreted by the researcher, related via the researcher's experience, and then expressed by the researcher. To minimize the amount of interpretation of the researcher and to maximize the metacognitive expressions of the student, Reynolds' (1993) design was applied where the student would attempt to solve the above problems (primary video) and then immediately watch themselves solving the problems with explicit instructions to explain their thought process (secondary video). Thus students worked with the researcher on the above problems for 30 minutes and then watched the video of their problem-solving process with the researcher for 30 minutes.

Results

Using mixed methods, two analyses were used; the first analysis was qualitative while the second was quantitative. The initial analysis involved multiple observations of all the videos using a phenomenological design and recording significant metaphors, problem-solving techniques, and all justifications. Moreover, distinctions between the student solving problems and student watching himself/herself solve problems was recorded.

The first analysis revealed that the students evoked metaphors rich in context and culture. For example, when working with P2, one student stated "Adding rational and irrational numbers is kind of like mixing oil and water." This perception dominated her problem-solving paradigm. Initially, it helped conceive of the question, but then raised complicated issues when deductive reasoning was needed. A more surprising result of the first analysis was the abundant use of the word "like". Students used the word "like" frequently demonstrating examples or counterexamples to guide their intuition. When trying to understand the problem, students used the word "like" for inductive reasoning rather than deductive reasoning.

There was significant evidence that a classification of metaphors (structural, ontological, orientational) was applicable to problem solving. Students were consistently able to discuss their problem-solving techniques as if their brain was a separate entity. In the secondary video, students evoked ontological metaphors personifying their mind as an entity from which they were analyzing. For example, one freshman changed from one question to another because they had to let their "subconscious work on it for a while". Another freshman stated "my mind plays games on me." The following is a list of conceptual metaphors (mainly structural) that the students related to their problem-solving strategies through metaphorical linguistic expression in the first analysis:

Table 1

Target Domain		Source Domain
PROBLEM SOLVING	IS	A JOURNEY, STRATEGIES, A HUNT, A BATTLE, A PRODUCT, A DESTINATION, A BUILDING, A GOAL, TRICKS, DISCOVERY
THE PROBLEM SOLVING PROCESS	IS	CONSTRUCTING, EXPERIMENTING, ILLUMINATING, TRAVELING, PLAYING, SEARCHING

The quantitative analysis had two parts. The first part attempted to verify Kovecses's (2010) and Lakoff's (2000) research in linguistics; that there is a hierarchy between structural, ontological, and orientational metaphors in mathematical problem solving. The second part attempted to verify that the frequency of the word "like" was related to the student's performance. It is important to note this voluntary pilot study included only nine participants and thus nine degrees of freedom which limited the study ($N<30$).

The first part of the quantitative analysis was calculated by counting the number of times each conceptual metaphor was used during the primary and secondary videos. The descriptive statistics are shown below:

Table 2

	Mean	Std. Deviation	N
T_struc	32.22	10.462	9
T_ontol	19.78	6.438	9
T_orient	14.78	7.870	9

MANOVA was performed on the data, and the Wilk's Lambda showed a strong significant variability between the conceptual metaphors ($F(3,9)=14.292, p=.003$ *with alpha*$=.05$). Additionally, the structural metaphors were most frequent ($\mu = 32.22$) followed by ontological metaphors ($\mu = 19.78$) and then orientational metaphors ($\mu = 14.78$) as was expected according to cognitive linguists.

The second part of the quantitative analysis demonstrated that the total number of times a student used the word "like" was related to their overall score. For each problem the student solved and could justify, the student was given a score of "1". For each problem the student solved, but could not justify, the student was given a score of ½. In this manner, students could receive an overall score between 0 and 3. There was a nearly-significant moderate negative correlation between the overall score and the number of times a student used the word "like" ($r=-.634$, $p=.067$). Additionally, there was a strong negative correlation to their score on P3 to the number of times a student used the word "like" in P3 ($r=-.937$, $p<.001$). Both of these results demonstrate that the more frequent the word "like" was in the student's explanation, the worse their performance in solving and justifying the problem.

Significance to Mathematics Education

This study's primary purpose was to develop and improve upon the use of conceptual metaphors for mathematics education. Three conclusions can be drawn from the results of this study. First, students are able to model and describe their problem-solving processes metaphorically. Students used personifying ontological metaphors to describe their mind as an external object and structural metaphors to describe how their minds interpreted problem solving and the problem solving process (Table 1). These conclusions offer insight into Q1, Q2, and Q3.

Secondly, the quantitative analysis showed that the linguist's cognitive hierarchy is tenable within mathematical problem solving as the student's metaphorical frequency was greatest with structural metaphors and least with orientational metaphors. While frequency doesn't guarantee the hierarchy alone, it strongly suggests future studies may find a relationship between the difficulty of a problem to prove versus problems to solve (Pólya, 1945) and the use of structural metaphors over ontological or orientational metaphors (Lakoff & Núñez, 2000). This led to uncovering better conclusions to Q4.

Finally, pedagogy is the core of mathematics education. Results that are not practical to the teacher limit their importance and generalizability. Student's use of the word "like" has relevance for many reasons. First, it demonstrates analogical reasoning, but in the inductive sense. As Lakatos (1976) demonstrates, inductive reasoning is necessary for deductive reasoning, but is not necessarily causal. Moreover, the negative correlation between frequency of the word "like" with the student's performance suggests that if students are unable to move beyond this inductive reasoning they will continue to hover around the problem unable to deductively reason its logical truth or falsity. For the teacher, this suggests that if a student is using the word "like" frequently in the same problem, they may be unable to conceptualize properly how to perceive the problem. Hence, if a teacher listens for the student's overuse of the word “like”, it may be an early indicator that the child is struggling with the concept or problem at hand. Understanding how students use the word “like” may improve student learning within mathematical problem solving (Q5).

Stating a metaphor alone is only a means of expression of experience. Yet if educators and researchers look to their application within mathematical problem solving, metaphors influence student learning. Moreover, this analysis has suggested ways in which conceptual metaphors can be used to improve teaching. Hopefully, this pilot study will lead to future studies encouraging and confirming that metaphorical conceptualization can aid mathematics educators.

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EFFECTS OF THE USE OF READING STRATEGIES IN A HIGH COGNITIVE DEMAND WORD PROBLEM, GRADES 6-8

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This ten-week comparison study investigated effects of incorporating reading strategies within a high cognitive demand warm-up problem on student learning, grades 6-8. Teachers opened class 3-5 days a week with a problem solving warm-up including three questions to focus on comprehending the problem. Post test means were significantly higher for the four 7th and 8th grade treatment teachers' classes. An open ended problem assessment showed the treatment more effective for high and low scoring students but little difference for the middle scoring students.

Problem solving has been a central theme for the mathematics curriculum, preK – college for many years. However, students in the United States continue to lag behind other nations in solving real world applications on PISA 2006. (Provasnik, Gonzales, & Miller, 2009). U.S. fifteen year olds' average score was in the bottom 25% of participating industrial nations, a position unchanged since PISA 2003. Problem solving difficulties may stem from deficiencies in math skills, conceptual understanding, reading skills, strategic knowledge, or student attitudes (Kroll & Miller, 1993). Improved problem solving achievement is critical for our students. This study investigates a classroom method to assist students with reading and understanding the problem context within a high cognitive demand word problem. Cognitive demand refers to the level and type of reasoning required by students to solve a problem (Stein et al., 2000). Problem solving is defined for this research as applying mathematics and critical thinking in new and novel settings in word problems.

Background

Research has shown that reading comprehension plays a key role in problem solving success. A meta-analysis of seventeen studies in middle grades mathematics (n = 4209) found a significant correlation, rho =.62, for reading achievement and math problem solving (Hembree, 1992). Recent studies have similar positive associations (Grimm, 2008, Vilenius-Tuohimaa et al., 2008). Grimm (2008) demonstrated that as early as third grade, reading scores were a positive significant predictor for 7th and 8th grade problem solving and data interpretation scores but not for math computation. Abedi and Lord (2001) found NAEP word problems modified for easier reading significantly improved student scores.

From 7th and 8th grade think aloud problem solving data, Pape (2004) investigated students' translation between text and forming a problem representation. Translation behaviors were categorized into Direct Transfer Approach students who transferred numbers directly without use of context and Meaning Based Approach students who recorded information, used the context, and showed understanding through explanations or justifications. The Meaning Based Approach students had greater problem success rates, fewer reading and math errors, and were more able to preserve problem structure upon recall. Now, research needs to focus on best classroom methods to foster this "Meaning Based Approach" to translation. This study seeks to assess an easy-to-implement self-questioning method for greater problem comprehension.

Polya (1945/1986) described the importance of reading comprehension as the first phase of his problem solving model: 1) understand the problem, 2) devise a plan, 3) carry out the plan and 4) look back. Charles and Lester (1984) found that effective instruction focused on Polya's model and problem solving strategies significantly improved middle grade students' problem solving achievement. The researchers recommended problem solving instruction should take place over time, use motivating tasks, and be conducted by teachers who modeling strategies.

A cognitive strategy can support students to think like a proficient problem solver. Montague and Bos (1986) found a seven phase cognitive strategy model successful with students with learning disabilities: 1) read, 2) paraphrase, 3) visualize, 4) hypothesize, 5) estimate, 6) compute, and 7) check. Relevant to this study are the 8th grade results from a recent "Solve It!" project applying this model (Montague, Enders, and Dietz, in press). In inclusive heterogeneous math classrooms, students used the strategies weekly within district materials and state sample word problems over a year. The Solve It! group (n = 319) had significantly higher growth on a test with ten one, two, or three step textbook type word problems and one higher level problem.

In the current study, middle grade teachers had been introduced to an easy to remember cognitive strategy model, ROPED: Read the problem, Organize the data, Plan, Execute the plan, and Does it work? (Crawford, 2005). With the first step, "R", the student reads the problem three times for: 1) overall meaning, 2) what the problem asks, and 3) information needed. Can embedding these reading strategies within a high cognitive demand word problem be an effective method to help students understand the context of the problem?

Methods

The treatment classes included two 6th grade (n =203), two 7th grade (n =178) and two 8th grade (n =186) classes (excluding Algebra I), in a rural, low economic middle school in the southeastern United States. At this school, 54% are classified as economic disadvantaged. Ethnicity is 57% Caucasian, 29% African American and 8% Hispanic. The comparison classes included two 6th grade (n =164), two 7th grade (n = 162), two 8th grade classes (n =133) (excluding Algebra I) in another middle school in the same district. In this school, 65% are classified as economic disadvantaged. Ethnicity is 42% Caucasian, 45% African American and 5% Hispanic. Teachers at both schools had been participating for two years with the authors in a Mathematics Science Partnership federal grant to improve student learning.

Warm-up word problems were written by one researcher to correlate with state math objectives in number, geometry and measurement, data analysis and algebra. Due to a need to increase learning in geometry, at least one geometry/measurement problem was included each week. Most problems were written to match the highest level of mastery from the state indicator applications, but not as detailed, in order to be completed in approximately ten to fifteen minutes. Problems were written to be relevant to student lives to create motivation (see Table 1).

Three questions were included with each warm up problem to focus on reading comprehension. First, students were asked: "Read the problem. What is the problem about?" The second question asked: "Read the problem again. What does the problem ask you to do?" and finally students were asked to: "Read the problem again and underline important facts for solving the problem." Occasionally, there was a variation in these questions such as "circle any words that you are unsure about".

Table 1.
Sample Problem Solving Warm-up for 7th grade

Your uncle tiles the hall floor that is 5 ft wide and 4 ft long. He uses 1 ft. square tiles. If the total cost of the tiles is $28.00, what is the cost of each tile? He tiles the bathroom floor next which is 6 ft by 8 ft with different tiles and pays $72. Are the bathroom tiles more expensive or cheaper? Explain your math calculations. Read the problem. What is the problem about? Read the problem again. What does the problem ask you to do? Read the problem again. Underline important information for solving the problem

Pretests were given at each school in mid September. In the following week, one researcher met with treatment teachers for a problem solving instructional session. Teachers discussed the three reading strategies, how to introduce these to students and the cognitive level of the first set of warm-up questions. Treatment classes began using the problem solving warm-ups to open class three to five days a week continuing for ten weeks. Teachers gave input weekly about objectives they preferred for the next problems by email or at a group meeting.

Teachers modeled the reading strategies with students, often asking for three student volunteers to read the problem aloud to the class. After each reading, students individually answered the comprehension questions. The researcher suggested omitting the reading questions after five weeks, but the teachers felt students needed the continued repetition. Completed problem sheets were collected at the end of each week. Researchers observed all teachers' classrooms, both treatment and control and met with teachers at each school every two weeks.

Data Collection and Analysis

A computerized pretest and posttest were designed for each grade from multiple choice word problems from ClassScapes database (http://www.classscape.org/ClassScape3/) to assess course objectives during the treatment. Sixth and seventh grade tests each had 32 questions and the 8th grade test contained 24 questions. This database, developed to correlate with state assessment word problems, is used by both schools for benchmark tests. Students took the post test at the end of the semester, one month after the end of the treatment. Data were analyzed using SPSS.

After ten weeks, an open ended assessment with two high cognitive demand word problems for each grade was given (Appendix A). Each problem was scored with a holistic rubric, levels 0 – 5 to assess how far a student progressed in the problem. For reliability, two researchers scored papers independently then discussed differences. In addition, from input from 6th grade teachers, problem 1 was rescored to accommodate two interpretations. Scores were collapsed into three categories: high scoring (5-4), middle scoring (3-2), and low scoring (1-0). Researchers agreed this would facilitate discussion and provide greater reliability.

Results

Pretests for both 7th and 8th grades revealed no significant difference in student learning between the treatment and comparison groups. For 7th, treatment mean = 47.6, SD =15.7 and comparison mean = 44.5, SD=14.4, p = .106. For 8th, treatment mean =35.7, SD=14.7 and comparison mean = 32.3, SD=12.5, p = .055. For sixth grade, there was a significant difference in the groups with treatment mean = 49.3, SD=14.1 and comparison mean = 41.9, SD=12.1, p <.001. Due to these initial differences in sixth grade groups, open ended assessment and posttest results will only be discussed for 7th and 8th grades. The problem solving warm-up group means were significantly higher on the posttest (Table 2).

Table 2.
Post-Test Equity of Mean Results

Grade	N	Mean % Correct	SD	t	p-value
6^{th} *					
PS Warm-up	198	61.32	18.68	2.91	.004
Comparison	163	55.73	17.48		
7^{th}					
PS Warm-up	168	52.89	18.47	5.18	<.001
Comparison	142	42.91	14.82		
8^{th}					
PS Warm-up	176	56.78	14.54	4.12	<.001
Comparison	138	50.34	12.66		

* 6^{th} grade results should be interpreted with caution

Table 3 provides the results for the open ended assessment. For both 7^{th} and 8^{th} grade, there is a significant difference in favor of the treatment group for the high scoring students (5-4) with Problem 1 pertaining to the number objective. For problem 2, the geometry problem, although the percentage of student in the PS Warm Up group is greater for high scoring students (5-4) at 7^{th} and 8^{th} grades, there is not a significant difference. The problem solving warm-ups appear to have a positive effect on students in 7^{th} and 8^{th} grades who are able to read and determine appropriate strategies to solve a complex problem.

For the low scoring students (1-0), there is a significant difference for both problems for grade 7. For 8^{th} grade students, the treatment group had a smaller percentage at the lower scoring level (1-0) for both problems, but not significant. The problem solving warm-ups appear to have a positive effect for the low scoring students at the 7^{th} grade.

For 7^{th} and 8^{th} grades, there was not a significant difference in favor of the treatment in the number of students who scored at the middle level (3, 2). These data reveal that the treatment did not have as great effect on the middle scoring students.

Discussion

The problem solving warm-up is an easy to implement method for teachers to open class

Table 3.
Open Ended Problem Assessment: Percent Scoring at each Level by Treatment

6th Grade	PS Warm-Up	Comparison	Chi-Square	P-value
Problem #1 **				
High (5-4)	16.6%	5.3%	10.72	.001
Middle (3-2)	45.1%	50.3%	.186	.666
Low (1-0)	38.3%	44.4%	.652	.419
Problem #2 **				
High (5-4)	26.9%	12.6%	10.20	.001
Middle (3-2)	30.4%	23.2%	2.13	.144
Low (1-0)	42.7%	64.2%	14.94	<.001
7th Grade				
Problem #1				
High (5-4)	28.0%	18.0%	3.876	.049 *
Middle (3-2)	33.8%	28.6%	.831	.362
Low (1-0)	38.2%	53.4%	6.93	.008 *
Problem #2				
High (5-4)	7.0%	5.3%	.359	.547
Middle (3-2)	44.6%	33.1%	3.816	.051
Low (1-0)	48.4%	61.7%	5.35	.021 *
8th Grade				
Problem #1				
High (5-4)	12.9%	3.1%	8.636	.003 *
Middle (3-2)	65.8%	70.3%	.652	.419
Low (1-0)	21.3%	26.6%	1.078	.299
Problem #2				
High (5-4)	14.2%	10.9%	.669	.413
Middle (3-2)	21.3%	18.0%	.487	.485
Low (1-0)	64.5%	71.1%	1.382	.240

* significantly different at .05 level ** 6th grade results should be interpreted with caution

each day with a high level problem and include reading comprehension strategies through self-questioning. Results indicate the reading strategies were beneficial for high scoring students (5-4). These students were then able to apply their reasoning, math concepts and skills to solve or almost solve the problem. Likewise, the emphasis on reading the problem allowed more treatment students to go beyond the 1-0 level. The students demonstrated a beginning understanding of the problem. However, students scoring 3-2 did not demonstrate they had the math concepts, skills or problem solving strategies to go further. Teachers at both schools need to embrace methods to develop reasoning and understanding of math concepts. Low scores on the geometry problems reveal the strong need for problem solving lessons (beyond a warm-up) with concept development in geometry.

The importance of translating words into an appropriate picture or math symbols appeared to be an important first step. The treatment groups had more students go beyond level 1-0 demon-strating a beginning understanding of the context through an appropriate drawing or symbol representation. Students reaching the 3-2 level had begun to apply the math concepts from the context. Teachers need to model how to draw appropriate representations for problem contexts based upon math concepts. In the Organize step of the ROPED cognitive strategy, students are asked to "close your eyes and image the situation, make a drawing or diagram and label information". Students can discuss, "how do you think up a plan and decide a strategy?"

From observing the two 7th grade treatment classrooms, teachers used the warm-up to ask students to share methods and talk about the concepts to further student learning. The warm-up was another classroom tool for formative assessment to uncover misconceptions. The achievement differences were the greatest for these teachers. At the 8th grade level, achievement differences between groups were less. The two comparison group teachers worked closely as a team to meet student needs, fostered student discussion of concepts and applied problem solving within instruction although not with the warm-ups. This may be a reason why their open ended assessment differences were not as great.

There is great concern about the high number of students scoring at level 1-0. Teachers reported that many of these students had severe reading difficulties. More strategies with graphic organizers to connect math vocabulary to concepts and symbols are greatly needed. Teachers need to utilize methods to identify words students do not understand then apply vocabulary strategies to build reading skills. Students can work in pairs to read problems aloud to each other, discuss the meaning and how to begin the problem.

Limitations

One limitation is that the improved problem solving may be due to the experience of solving more high level problems and not from the reading strategies. A follow-up study can use a treatment group with the reading strategies, a group with the problems only and no reading strategies and a comparison group. Also, school based research has many difficulties. There are problems with validity that researchers are working to address with further data analysis with sixth grade data. Another issue arose with the pretest in the study. Excess time was required for students to learn how to log into the database. Not all students were able to complete the pretest. This happened at both schools. The small number of teachers in the sample is a limitation.

Conclusion

This research suggests that embedding reading strategies within a high cognitive demand word problem has promise as an effective method to increase learning in the middle grades math. The problem solving warm-up with self-questioning is an easy to implement classroom method. However, teachers need to go beyond a mere warm-up and include problem solving application lessons and teaching concepts through problem solving.

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APPENDIX A

Open Ended Assessment

Grade 6:

Problem 1

Anna's aunt has agreed to take Anna and her three cousins bowling. The prices at Ed's Alley are $4.50 for adults per game, 4.00 for children 12 and over per game, $3.50 for children under 12. They spend $19.80 for dinner and everyone bowls two games. One cousin is 15, Anna is 11 and her other cousins are under 12. What is the total cost of just the bowling (without dinner)? If they share the cost of the dinner equally, how much is Anna's part for the bowling and the dinner?

Problem 2

Mr. James is planting a rectangular vegetable garden. He decides that he wants the area of his garden to be 180 square feet. He measures 10 feet for the width of the garden. What is the perimeter of his garden? If the cost of fencing is $2.10 per foot, what will be the cost of the fence around the garden?

Grade 7

Problem 1

A group of 7th graders go to the Pizza Hut after the basketball game. They purchase 6 large pizzas. Each person eats 2/5 of the pizza. The cost of each pizza is $10.50. How many people can eat pizza and how many pieces will be left? What is the cost per person for the pizza? Show all your work.

Problem 2

Last spring Zack planted a square garden with width 7 feet. His neighbor's garden was also square but was four times the area of Zack's garden. What is the width of his neighbor's garden? What are the perimeters? What is the ratio of the perimeters of the two gardens?

Grade 8

Problem 1

Last Friday night, Tim and four friends go to the Pizza Hut for dinner. Tim buys a drink for $1.75 and ten medium buffalo wings for $7.99. He and his friends share 2 extra pizzas. Tim spends a total of $13.94. What is the cost of one pizza?

Problem 2

Mrs. Watson is painting the walls in her living room. One wall is 20 ft by 9 feet but has one window that is 6 feet by 5 feet. The opposite wall is 20 feet by 9 feet but has a door opening that will not be painted that is 6 ft by 7 feet. Two other walls are 13 feet by 9 feet. She goes to Lowes and finds that one gallon of paint costs $18.50 and covers approximately 400 square feet. If she knows she needs two coats of paint because of the color, what will be the cost of the paint?

WORKING TOGETHER: STUDENT ENGAGEMENT IN A MIDDLE SCHOOL MATH CLASSROOM

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Students may benefit from working in small groups when each individual is engaged, at least to some extent, during the problem solving session. This paper focuses on the types of engagement that emerge as a group of middle school students work on a conceptually challenging mathematics task. The students' engagement and work are discussed with respect to cognition and affect, both critical to a student's mathematical success. Classroom examples are provided, including student explanations of their mathematical ideas and reasoning, as well as questionnaire responses regarding student interactions and engagement.

Student engagement in the classroom contributes to developing students' confidence in their abilities to learn mathematics (National Research Council, 2002). Thus, student affect and motivation may facilitate as mathematical engagement and successful problem solving (e.g., DeBellis & Goldin, 2006). In this study[1], I explore the cognitive and affective engagement of middle school students investigating a complex mathematics problem. Students' interactions are characterized from social, cognitive, and affective perspectives, as they work through a conceptually challenging task. After describing the theoretical framework, I explain the methodology and the task given to the students. Next, classroom episodes and questionnaire results are presented to illustrate how the theoretical framework was applied. Finally, I discuss results and some limitations of this study.

Theoretical Framework

The framework for this paper is based upon the work of Goldin, Epstein, and Schorr (2007), who recognized a need for a new construct to describe the dynamic interactions they saw in previous studies focused in mathematics classrooms (Alston et al., 2007; Epstein et al., 2007). The construct, called an **engagement structure**, describes an idealized, recurring, highly affective pattern inferred from observed behavior (Epstein et al., 2010; Schorr, Epstein, Warner, & Arias 2010a,b). An engagement structure may become active for an individual in a particular social context such as a mathematics classroom. These structures take into account the complexity of a student's cognition, behavior, and affect when they engage with mathematics problems. Each engagement structure includes a motivating desire or aspiration to engage in some activity, followed by an action aimed at achieving his or her motivating desire (Schorr et al., 2010b). Epstein et al. (2010) have identified and described several engagement structures. In this paper, I focus on two related structures: "Look How Smart I Am" (LHSIA) and "Let Me Teach You" (LMTY). These two structures have different motivating desires and potential resulting actions but both involve sharing information about a mathematics problem.

LHSIA may occur when a student experiences a motivating desire to appear smart, impress others, or "show off" her mathematical ability, knowledge, or intelligence (Rossman, Schorr, & Warner, 2010; Schorr et al., 2010b). A sense of satisfaction may occur when others acknowledge this individual. While the student's classmates may recognize her as intelligent or knowledgeable, another potential reaction may be to ignore or reject this student's ideas. The student who feels rebuffed may become defensive of either her ideas or herself.

In a different scenario, the student attempting to impress others may find that her ideas are considered valuable. However, a classmate may express confusion or request clarification, prompting the individual to further explain her ideas or strategies. LMTY may become active for this student, particularly if the individual (the tutor) experiences a motivating desire to teach or explain an insight, concept, or strategy to another person who appears to not have this understanding (the tutee) (Rossman et al., 2010; Schorr et al., 2010b). Ideally the tutor is successful in communicating her ideas and may feel a sense of satisfaction from helping someone.

To facilitate analysis of the structures beyond drawing inferences from observed behavior, the senior researchers designed a questionnaire for students to complete (see Epstein et al., 2010). Students were asked about different ways they were engaged during class, their motivating desires, and actions taken to satisfy those desires, among other pertinent items (Rossman et al., 2010). The items which may indicate either LHSIA or LMTY can be found in Tables 1 and 2. The next section describes the data that was collected and the task given to the students.

Methodology

The students who participated in this study live and attend school in a large urban district with low-income families and a large minority population. These students were encouraged by their teacher to explore conceptually challenging tasks while working in small groups, where they could share solutions, ideas, and explanations with one another.

I and a team of graduate student researchers observed this class over three consecutive school days during which they worked on the task described below and presented their solutions to one another. Groups of 3 or 4 students were created using a random number generator, and each group was recorded via video and audio. Researchers did not interact with the students, aside from occasionally providing supplies. Each session concluded with students completing the questionnaire independently, while a researcher read the directions aloud. The video and audio recordings, as well as questionnaire responses, were analyzed for evidence indicating LHSIA or LMTY. The qualitative findings based on the students' behaviors were compared to the questionnaire responses to confirm or refute inferences based on observations.

The Building Blocks task described below, adapted from our pilot study[2], was given to the students in this class. It was selected for its likelihood to be conceptually challenging to most participating students based upon criteria cited in Stein, Smith, Henningsen, and Silver (2000). Each student received a task sheet with instructions and the diagram given in Figure 1.

I was constructing towers as you see below. I noticed that each time I made the tower higher, I had to add more blocks on the sides to stabilize the structure. I would like to know how many cubes I will need to build a 5-block high tower and a 10-block high tower. Generalize, if you can, on how many blocks I will need for any size tower?

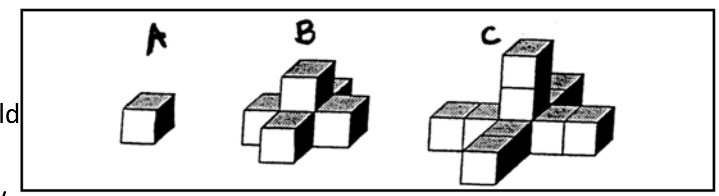

Figure 1: The Building Blocks Task

The ultimate goal of this task is to determine an algebraic formula or model to represent the number of blocks needed in a tower of a given height. During the class sessions, students were encouraged to draw sketches, create tables, and use plastic interlocking blocks to build the towers. These activities provided opportunities for discovering emerging patterns and solution strategies.

Findings

In this analysis, I examine how two engagement structures may have become active for one group of students working together: two boys, Manny and Damon, and one girl Deanna[3], the quietest member of the group. According to their questionnaire responses, all three students enjoyed working together on the task. The transcript below starts shortly after the class starts and uses line numbers to represent speaking turns.

After the teacher finished introducing the task, Damon was the first in his group to share his ideas about the problem, targeting Manny as his primary audience. The boys briefly discussed the 100-block high tower, and, in the transcript below, Manny begins to focus on the three towers seen in Figure 1. He seems to be developing his own understanding of the task before explaining the construction of the towers to Deanna. Deanna quietly listens to her classmates.

19 Manny: (to Damon) So if it's 30 then you add 1, that's 4, that's 5 more.
(shows 4 on his fingers)
20 Damon: So for 100 it would be 99 on each side.
21 Manny: Wait up. Wait up.
22 Damon: For 100, there would be 99 on each side! (said with more emphasis)
23 Deanna: I don't get it.
24 Damon: You don't get what I'm saying? (Deanna shakes her head and is smiling)
25 Manny: Wait up. Wait up.
26 Damon: (using the tip of his pen to point to the diagrams on Manny's paper)
If you're saying that if it's 2 blocks high and 1 on each side every time I go up 1, this will go up 1. So that's what I'm saying it's going to be. One hundred on 99. A hundred.
27 Manny: So it's like when you add 1, you add 4.
28 Damon: Yeah.
29 Manny: So you add 5 at a time. (looks at Deanna, including her in the conversation)
30 Damon: Yeah.
31 Manny: Alright. So let's do that. One more is 5 more, which is…
(looks over to Deanna while speaking; she smiles and shakes her head, indicating that she does not understand their strategy)
Deanna doesn't understand.
32 Damon: She knows what I'm saying.
33 Manny: (reaches over to Deanna's desk; uses his pen to point at her paper while she is looking on) Okay. One block you got zero. (pointing to figure A)
34 Manny: Okay. Then he added one more, one more to each side.
(points to figure C; Deanna nods her head slightly)
35 Damon: Every time you add 1 to the…(overlapping with Manny) to the height?
(He and Manny are pointing to Deanna's paper)
36 Manny: Every time you make it 1 higher, it adds 5 blocks.
37 Damon: Yeah.
38 Manny: One, two, three, four, and five.
(pointing to Deanna's paper to help explain his point)
39 Deanna: Oh. (smiling; brings paper closer to her)
40 Damon: Get it? (emphatically)
41 Deanna: Yes!! (sounds exasperated with Damon)

Throughout the episode above, Damon continuously tries to contribute to the conversation, speaks with an assertive tone of voice as he discusses the 100-block high tower, and often interjects, “Yeah” while Manny is speaking. His behaviors, such as repeating what others have said, indicate that he wanted others to recognize he already understood the ideas Manny was explaining. These behaviors and tone of voice suggest he was motivated by a desire to show his classmates that he is smart and possesses knowledge about the task. Therefore, Damon may have an active LHSIA structure. Damon’s responses to some questionnaire items (Table 1) suggest that he was motivated by the desire to appear “smart” to others and that he took actions to try to impress others with his ideas. Damon responded that he did not wish to show off, which combined with his open-ended response that he was “excited to be videotapes while I’m showing what I capable of doing [*sic*],” suggests that Damon possibly believed that showing off is different from impressing others with his mathematical ability.

Damon may have also activated the LMTY structure in this episode. Though Damon often did not explain his ideas, one exception occurred when he tried to explain why there are 99 blocks on each of the sides or legs when the tower has a height of 100 (a mathematically correct idea). When he attempts to draw a connection between the 2-block high tower and the 100-block high tower in turn 26, he appears to be motivated by a desire to share and explain his ideas. On the questionnaire (Table 2), Damon responded that he wanted to help others understand the mathematics, and indeed he may have perceived his actions as helpful.

Within this same episode, Manny fosters a different kind of interaction with Damon and Deanna. He takes time to explain the problem to Deanna, starting at turn 31. It appears that the LMTY structure is active for Manny when he explains that each tower is constructed by adding five blocks to the previous tower, referring to the towers given in Figure 1 as examples. Manny appears to be motivated to help Deanna because he continues with his explanation until Deanna indicates an increased level of understanding by smiling as she says, “Oh.” On the questionnaire, Manny responded that he “sometimes” helped his classmates to understand the mathematics. He replied that he enjoyed teaching others and that he gave helpful suggestions.

In addition, Manny’s positive responses to some of the LHSIA questionnaire items suggest that he may have wanted to appear smart. Prior to helping Deanna, Manny exchanged ideas about the task with Damon and fostered an understanding of the task within the group. Manny may have felt smart and believed he already had a sufficient understanding of the task when Deanna expressed confusion. His motivation may have changed from appearing smart to helping a classmate, thereby shifting from an active LHSIA structure to an active LMTY structure.

Table 1.

Questionnaire Items Which May Indicate LHSIA structure

Questionnaire Items – Statements	**Damon**	**Manny**	**Deanna**
I wanted people to think that I'm smart.	Sometimes	All the time	All the time
I wanted the teacher to think that I am a good student.	Sometimes	Never	All the time
I tried to impress people with my ideas about the problem.	All the time	Sometimes	All the time
I felt smart.	All the time	All the time	All the time
People seemed impressed with the ideas I shared about the problem.	Sometimes	Never	Sometimes
People saw how good I was at the math today.	Sometimes	Never	All the time
Thoughts (Yes/No; Hardly ever/Sometimes/Often)			
I want you to know just how smart I am.	Yes	No	Yes
I wanted to show off.	Hardly ever	Hardly ever	Hardly ever

Table 2.

Questionnaire items which may indicate LMTY structure

Questionnaire Items-Statements	**Damon**	**Manny**	**Deanna**
I wanted to teach another student something that I knew that the other student did not know.	All the time	Sometimes	Sometimes
I helped someone see how to do the math.	Sometimes	Sometimes	Sometimes
I listened carefully to the ideas of someone I was trying to help.	All the time	Sometimes	All the time
I gave helpful suggestions.	Often	Often	Often
Others listened carefully to my ideas.	All the time	All the time	Sometimes
Thoughts (Yes/No)			
I like teaching this person things that I know.	Yes	Yes	Yes

Discussion

Engagement structures, including LHSIA and LMTY, may become active for any individual and no one person is limited to a single set of active structures. In fact, one individual may activate multiple engagement structures in a class period, as we see with Damon and Manny, based on questionnaire responses and inferred behaviors in this segment. Deanna did not exhibit either the LHSIA or LMTY engagement structure in this episode, though her behaviors at other times during the class session may provide evidence of an active LHSIA or LMTY structure. Deanna appeared to be cognitively engaged throughout the episode, as revealed by her behaviors, such as continuously looking at Manny while he spoke and shaking her head to indicate either agreement or confusion. Several other engagement structures have been described, so at least one of those is likely able to describe Deanna's engagement throughout this episode, but that is beyond the scope of the analysis for this paper.

Though both boys may have activated both LHSIA and LMTY, in this particular episode, I infer from the students' behaviors and tone of voice that Damon was more inclined toward an active LHSIA structure and Manny exhibited an active LMTY structure as he helped Deanna. The context in which these structures were activated must be considered as well. Damon's enthusiasm surrounding this task may have encouraged his desire to impress others with his knowledge. Manny may not have tried to help Deanna if she had not stated that she was having difficulty understanding the task. Her sustained attention encouraged Manny to persist with his explanation until she expressed her understanding.

This paper reports on an exploratory phase of the study, and more work is yet to be done. One goal for this study is to describe and differentiate the LHSIA and LMTY structures. By identifying a set of characteristics for each, I aim to better understand the moment-to-moment student engagement as it occurs in the classroom. Much of this analysis is drawn from the questionnaire, which is a static instrument given at the end of the class session. As it is meant to measure dynamic interactions, the students' responses may not always correspond to the researcher's inferences. Additional qualitative and survey analysis will be conducted on all the groups who participated in this study, allowing for further examination of questionnaire responses as well as further exploration of the characteristics of these two and other engagement structures.

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[3] Pseudonyms are used to protect the identity of study participants.

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A STANDARDS-BASED SYSTEMIC APPROACH TO ELEMENTARY MATHEMATICS: ELEMENTARY MATHEMATICS CLINICS AS AN INTERVENTION

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Proficiency in mathematics has become an increasingly important goal. Research in mathematics education reveals that gaps in students' understanding appear early. These gaps grow rapidly as students progress through their schooling. The end results for these students are poor mathematical achievement and the pruning of multiple future career pathways. This paper will reveal the efforts of an elementary mathematics clinic that was developed around the elements of a standards-based system. The grant funded clinic was designed to support students in their current mathematics curriculum and also identify and reduce gaps in their mathematical understanding.

There exists an urgency to improve mathematics education in the United States. Numerous studies have revealed students from the United States are not the highest achievers in mathematics on international comparisons (Stigler, 1999; Rutherford, 1990; National Mathematics Advisory Panel, 2008). Increasing mathematical achievement is a priority for the national security of the United States (National Center on Education and the Economy, 2007), and this security is based on all students achieving mathematics literacy to remain competitive for future employment. Mathematics literacy is essential in the 21st century global economy because science, technology, engineering, and mathematics (STEM) fields are among the fastest growing economic sectors.

The United States cannot maintain its global economic advantage unless it can supply the demand for workers in STEM fields (National Academy of Sciences, 2007). The United States Department of Education (2006), revealed, “More than half of the undergraduate degrees awarded in China are in the fields of science, technology, engineering and math, compared to 16 percent in the U.S.” The response to this threat has resulted in the introduction of standards-based instructional systems.

Standards-Based Instructional System

The elements of a standards-based instructional system are clear standards, fair assessments, curriculum, instruction, resources and materials for instruction, and interventions (i.e., safety nets) (Marsh, 1999; Tucker, 2002; National Center on Education and the Economy, 2007). The focus of this study explores one intervention related to students' mathematical achievement. The effort described in this study, to improve struggling students' achievement in mathematics, was approached using a standards-based instructional system model.

The theory behind the model is that improvements to students' academic achievement occur when all elements of the system are aligned and cohesive. Attention only to one element (e.g. curriculum) may not produce desired results in achievement. For example, mastery of Algebra is often cited as a gatekeeper to academic success in high school and beyond (Usiskin, 2005). A mandate for all students to complete a traditional Algebra course by the end of eighth grade is not likely to prepare more students for higher-level mathematics courses in high school. Instead, the number of students failing Algebra is likely to increase because the other elements of the system were not addressed, not to mention consideration of such students having the prerequisite knowledge and skills for success in Algebra. All of the elements of a standards-based instructional system were addressed during this study; however, what follows reveals only the element of intervention.

Intervention

The intervention at the center of this study is an after-school mathematics clinic for and fourth grade elementary students. Elementary students were selected under the premise that interventions work best when applied early, before the achievement gap grows too large (Hill, 2003). The model for the elementary mathematics clinic was to support students in their regular curriculum (e.g., homework help) and identify and eliminate their learning gaps through targeted instruction.

The identification and elimination of learning gaps was facilitated by the use of a web-based program called ALEKS, or Assessment and LEarning in Knowledge Spaces, and is based on Knowledge State Theory (Falmagne, 2008). Unlike interventions that rely on vast databases of test items, ALEKS identifies gaps in students' mathematical achievement and targets the correct skills and processes that students are ready to learn next.

Such identification and targeting aligns with best practices for increasing math performance for students below grade level (Burns, 2007). Further, computer-assisted instruction has been proven effective for increasing fluency in students with learning disabilities

(Goldman, 1997). An intervention may be declared a success if students return to grade-level mathematical achievement before they leave elementary school.

Methodology

The participants in this study were 27 fourth grade students from Title I elementary schools in a suburban school district in northeastern Pennsylvania. Students were identified for voluntary participation in the after-school elementary mathematics clinics based upon teacher recommendation or by not scoring proficient in mathematics based on the Pennsylvania System of School Assessment (PSSA).

The clinic ran twice a week from September to March. Each clinic was approximately one hour in duration and staffed by a team of certified teachers (i.e., the math coaches) using a seven to one student teacher ratio. Each student was enrolled in an ALEKS course at one grade level below their current grade level (i.e., fourth grade students were entered into a third grade course). At the first meeting of the mathematics clinic, the students completed an assessment by ALEKS to determine what they already knew and what they were able to learn based upon their knowledge state. After this initial assessment, students were to spend at least 50% of clinic time using ALEKS.

A typical clinic meeting began by helping students with their regular in-school mathematics curriculum homework or playing math games to increase their mathematical fluency. The second part of the clinic was spent engaged with ALEKS or receiving direct instruction from a coach. The instruction was targeted, based upon diagnostic information from ALEKS. For example, the program may reveal that eight students are ready to learn ‘estimating a product.’ The coach then would provide instruction to those eight students on that topic while the other students worked individually on the program at their computer.

The measurable designed outcomes were to (1) identify deficiencies in students’ mathematical achievement using five categories; these were *Number & Operations*, *Measurement*, *Geometry*, *Algebraic Concepts*, *Data Analysis & Probability*, (2) calculate rates of students’ mathematical achievement to formulate the time needed for each student to attain grade-level proficiency, and (3) evaluate academic achievement and growth of participants using PSSA and Pennsylvania Value-Added Assessment System (PVAAS) data.

Findings

Deficiencies in Mathematical Achievement

Fourth grade students who participated in the math clinics were placed in a third grade ALEKS course. The first time students logged onto ALEKS, they were assessed by the program to identify what they already knew about third grade mathematics. The initial assessment revealed severe gaps in the participants' understanding of geometry, algebraic concepts, and data analysis and probability (Table 1). Clinics A and B represent students from two separate elementary buildings.

Table 1.

The data above reveal the number of standards mastered during an initial assessment (i.e., subscript I) and final assessment (i.e., subscript F). The numbers in parentheses indicate the number of standards in each reporting category.

	Number & Operations	Measurement	Geometry	Algebraic Concepts	Data Analysis & Probability
Clinic A_I	7.3	0.9	0	0	0
Clinic A_F	11.4	3.1	0.3	1.4	1.1
Clinic B_I	5.3	0.5	0	0	0
Clinic B_F	10.4	2.5	0.6	1.5	0.5
	(17)	(7)	(5)	(6)	(4)

One report available from ALEKS is the identification of the number of standards mastered by students. The third grade ALEKS course is aligned to 39 mathematics standards divided into five reporting categories. A standard is considered mastered when a student has mastered at least 70% of the topics for that standard.

The data revealed that students had achievement gaps in all five reporting categories. The initial assessment data showed that students had no mathematical understanding of the standards other than the reporting categories number and operations and measurement. Although these are aggregated data, it was possible for the math clinic coaches to analyze an individual participant's data. Hence, it was possible to identify mathematical deficiencies that were informative for planning either group or individual instruction. For example, if 73% of the participants in the clinic had not yet mastered the standard *differentiate between even and odd numbers*, the coach may call those students together for targeted instruction on that topic while the other students worked independently on ALEKS.

Intervention Time

The participants in the elementary mathematics clinics met for one hour twice a week after the end of the regular school day. The clinic opened in late September and closed in late March; based on available funding for the program. Data from the mathematics clinics provided information about how much intervention time was needed for participants to return to grade level.

The range of hours spent on ALEKS during the clinic was 6.4 hours to 21.8 hours. After engaging with the software for seven hours, the program calculated how much additional time was needed for an individual to complete the program's course (e.g., third grade mathematics). The range of predicted time necessary to complete the ALEKS course was 5.1 hours to 60.5 hours. There was not a statistical relationship between the hours spent on ALEKS and the hours needed to complete the course. The lack of a relationship may indicate that participants in the clinics had different gaps in their mathematical achievement and learned at different rates.

The findings of this study may influence the design of future elementary mathematics clinics with the elimination of time as a constant. The sum of the hours spent on ALEKS and the predicted hours needed to complete the course reveal that the majority of the 27 participants needed between 6.4 hours and 37.5 hours of intervention to return to grade level. In contrast, one quarter of the participants needed over 37.5 hours, the upper extreme needed 80 hours of time on ALEKS to return to grade level.

The question of how much time to devote to an intervention requires a balance between the urgency to close achievement gaps and the realistic amount of time a student can maintain meaningful cognitive engagement. Another consideration is that students with large achievement gaps may benefit from interventions prior to fourth grade. The challenge with considering earlier interventions is that the earliest the PSSA is administered is in third grade. By the time assessment results are released, a struggling student has already completed third grade.

Achievement and Growth

The fourth grade PSSA mathematics is perceived as a more rigorous exam than the third grade exam. A reason for this perception may be illustrated using PSSA data from the participants in this study from their third grade exams and fourth grade exams (Figure 1). The maximum raw score on the third and fourth grade PSSA mathematics assessment is 72. A comparison of the participants' third and fourth grade PSSA mathematics average raw scores shows a decrease by 24%, from 55 to 42. In contrast, the participants' average scaled scores increased by 10%, from 1218 to 1346. These trends were not unique to the participants in this study. The average raw score for all students in the district between third and fourth grade was decreased by 17% and the average scaled score increased by 9%.

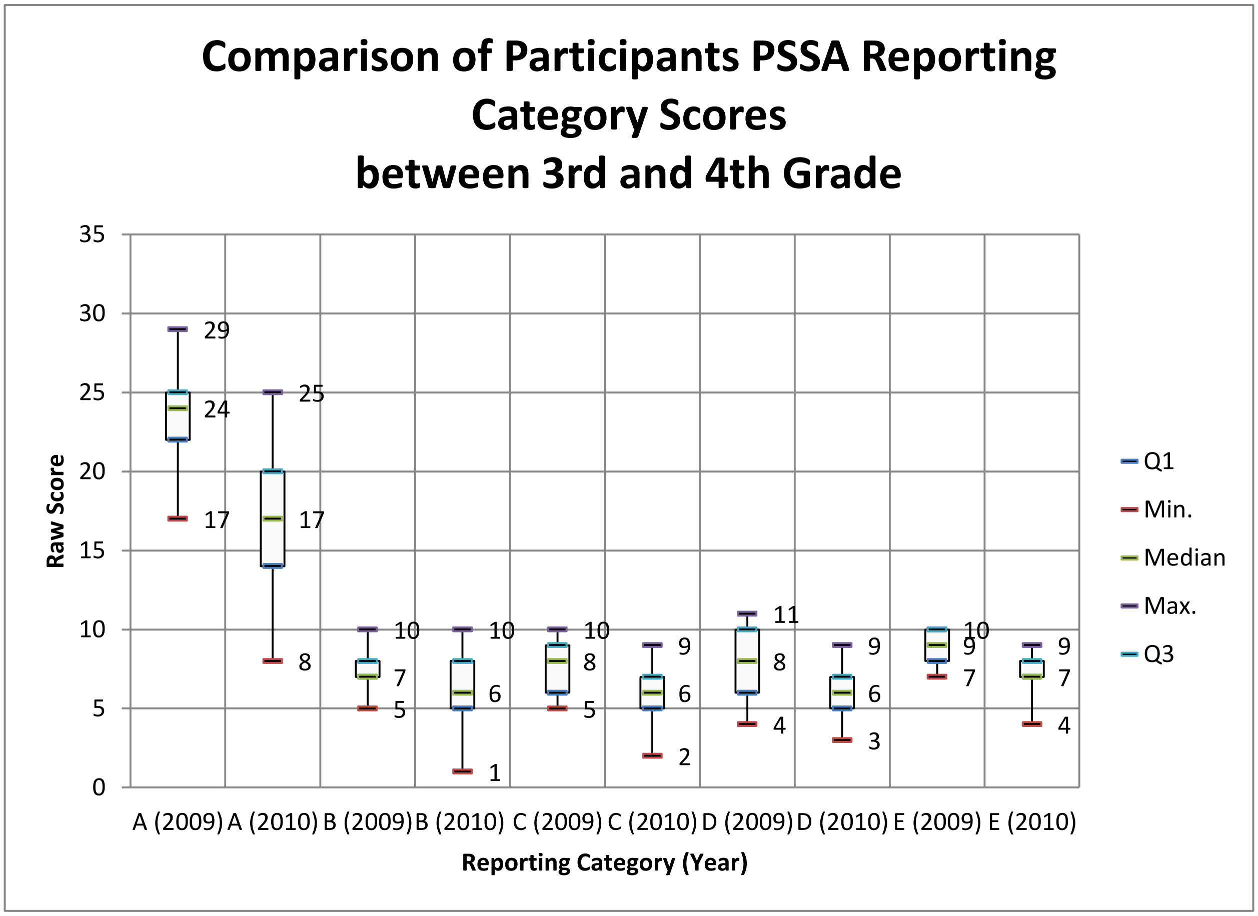

Figure 1. PSSA mathematics data from the elementary mathematics clinics suggest different levels of rigor between the third and fourth grade assessments. The categories are: (A) Number and Operations, (B) Measurement, (C) Geometry, (D) Algebraic Concepts, and (E) Data Analysis & Probability. The unconventional order of the statistical points in the legend is a manifestation of the spreadsheet used to construct the boxplots.

The five statistical points used to construct boxplots are a useful way to compare the participants' performance between the third and fourth grades. There is a decrease in raw score in almost every case. For example, the median raw score for reporting category C (i.e., measurement) decreased by 2 points between 2009 and 2010.

A participant's performance on the PSSA math is not measured by raw score. Instead, scaled scores are utilized and the level of performance is determined by cut scores. The

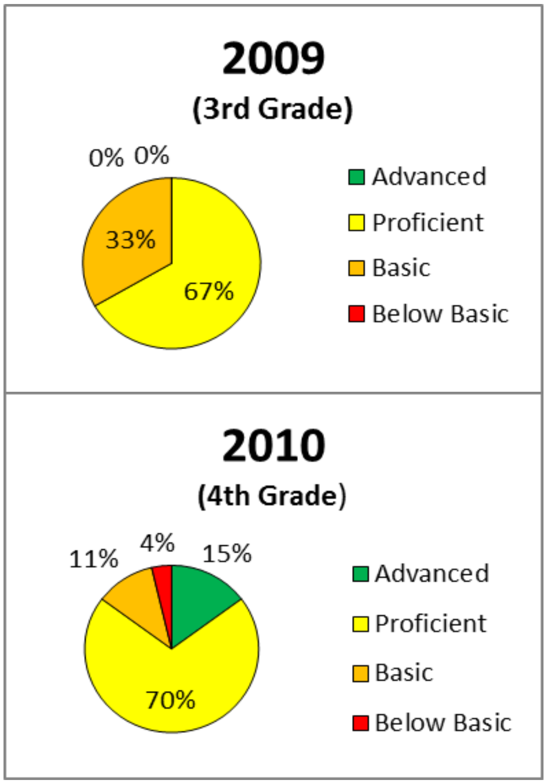

Figure 2. The participants in the math clinic had a greater percentage of proficient or better on the more rigorous fourth grade PSSA mathematics assessment than their third grade results.

proficient cut score on the PSSA math when the participants of this study were in third grade was 1180. The fourth grade cut score was 1246. A comparison of performance level depicts an increase in participants' achievement (Figure 2). For example, 67% of the participants were proficient in third grade whereas 85% of the participants were proficient or better on the fourth grade assessment.

A final way to evaluate the effectiveness of an intervention is through a custom diagnostic report using Pennsylvania Value Added Assessment System (PVAAS) data. A custom diagnostic report reveals the mean gain for subgroups of students. Subgroups comprised low, middle, and high achieving participants. One clinic was too small to analyze because a custom diagnostic report requires at least fifteen data. The mean gains of the clinic that was analyzed were -3.4 (low), 5.6 (middle), and 6.6 (high). The standard error on the low group extended across the zero mean gain reference line; this suggests the results are not significant. A zero gain may be interpreted as a student learning one year of material over one academic year. A positive gain may be an indication that a student gained more than a year's worth of material. The PVAAS data revealed that at least two-thirds of the participants in the math clinics had academic gains.

Conclusion

There is a clear urgency to improve elementary mathematics in the United States. One start to this reform is to catch students who struggle in mathematics early. Properly designed interventions, or safety nets, can catch students when they fall and return them to where they should be. Measuring the impact of an intervention designed to reduce gaps in students' understanding of mathematics is a challenge. Perhaps the best way to measure is through multiple lenses (e.g., internal and external measures).

The participants in this study were fourth grade students that were identified as struggling in mathematics. The participants were invited to attend a voluntary after-school mathematics clinic that was designed to support them in their current grade level mathematics and also identify and reduce any gaps in their understanding of mathematics. The participants in this study experienced an average increase of 128 points to their scaled scores on the mathematics PSSA.

Appropriate data have become an increasingly important tool to meet the challenges of high-stakes testing. The data from this study suggest that the participants in the math clinics did gain academically. What the data cannot reveal is a direct correlation between any specific component of the intervention (e.g., ALEKS) and student achievement. Rather some other parts of the intervention, or parts working symbiotically, worked for these participants (e.g., the coach, lessons selected, etc.). Perhaps the findings of this study may be useful to further improve the clinic model to serve as an even better safety net for students.

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BETTER MATHEMATICS THROUGH LITERACY: BUILDING BRIDGES FOR MEANINGFUL LEARNING

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This paper presents the concept and structure of the Better Mathematics through Literacy project and its benefits in leading early childhood classroom teachers and intervention specialists in adopting a student-centered and literacy-based approach to mathematics.

Over the past decades, mathematics teacher educators and our educational partners have engaged in an on-going conversation about the critical importance of teaching and learning mathematics with understanding, especially within the early grades of school (National Council of Teachers of Mathematics [NCTM], 2001; NRC, 2001). These formal learning experiences set the tone and expectations in the mind of young children for what it means to know and to do mathematics. Standards documents from professional organizations, specialized reports from national-level commissions and advisory panels, and renowned experts in our field have challenged mathematics education professionals in colleges and universities responsible for preparing preservice elementary teachers and educational leaders responsible for designing and delivering professional development experiences for inservice elementary teachers to critically examine how teachers and students come to view, to know, and to understand the mathematics they are expected to learn (NCTM, 2007; National Mathematics Advisory Panel [NMAP], 2008).

These advocates of reform in the approach to the mathematics of early childhood consistently advocate for using problem-based, constructivist approaches to mathematics that encourage the formulation of ideas and concepts through discovery and inquiry and the use of classroom discourse and reasoning to communicate mathematical thinking and sense-making. Yet, daily, within the context of early childhood classrooms throughout the United States, a large majority of teachers do not have conceptual understandings of the mathematics they teach to effectively support and structure the pedagogical strategies advocated within these reform documents (Ball, Hill, & Bass, 2005; Ma, 1999). Further, many inservice and preservice early childhood teachers feel they lack the pedagogical skills to successfully implement mathematics instruction that falls outside the predominant tell-show-do framework that permeates our nation's mathematical landscape. Thus, many teachers challenged with establishing a well-connected, conceptual, and integrated foundation for learning mathematics in the minds of young children that is predicated on purposeful problem-solving, reasoning, and communication often default to a teacher-centered approach that relies heavily on the memorization of isolated facts, the repeated implementation of canned algorithms that have no inherent meaning, and mathematics classrooms that function within a framework of sanctioned silence (Boaler, 2008; Van de Walle & Lovin, 2008).

In an attempt to implement the vision of meaningful early childhood mathematics within a small section of Appalachia, the Better Mathematics through Literacy (BMTL) project has been designed as a one-year professional development experience for inservice early childhood teachers and intervention specialists. Our main goals of the project are to strengthen conceptual mathematical content knowledge and to examine holistic approaches to mathematics through engaging, learner-centered activities, structured classroom discourse, the infusion of the NCTM Process Standards and the literary devices of writing, reading, and communicating (Burns, 1995; Kenney, 2005; O'Connell, 2005; Storeygard, 2009). This purpose of this paper is to share the structure and approach of the Better Mathematics through Literacy project to the wider mathematics education community and to demonstrate the impact and implications of the project experience on our participants' classroom practice as a viable pathway to teaching and learning mathematics with understanding.

Theoretical Framework

Mathematics reform efforts have been an attempt to move instruction away from the tradition in which mathematical knowledge is viewed as stoic, sequential, discrete, and easily understood by students through a public display of symbolic manipulation (Draper, 2002) and toward an instructional approach in which mathematical knowledge is viewed as an individual construction in the mind of a learner as he or she interacts with people and things in the environment. The National Council for the Education of Young Children [NAEYC] and NCTM issued a joint statement to issue a position that advocates for a high-quality, challenging, and accessible mathematics education for three to six year old children (2002) that is predicated on the active engagement of student thinking through exploration and the articulation of developing mathematical ideas and thinking as it naturally arises in the context of the students' investigation of real-life problems (Lee & Ginsburg, 2009). Yet, current classroom-based research on mathematics instruction in early childhood settings demonstrates that even though teachers have the best intentions to provide best practices to young children, many are still confused and anxious about constructivist approaches to the teaching and learning of mathematics and hesitant to change (Lee & Ginsberg, 2007a, 2007b).

The BMTL project was built on a framework of literacy and language instruction in which most early childhood teachers feel comfortable (Varol & Ferran, 2006) and competent. By this we mean that early childhood teachers are adept in getting students immersed and engaged in approaches that are constructivist in nature with respect to language and literacy learning. In the design and implementation of the BMTL project, the design team made specific and purposeful attempts to parallel the workshop instruction around Cambourne's (1998) Conditions for Learning which have been found to be critical features of classrooms in which young children are led to language acquisition and fluency. These core principles from Cambourne that are manifest in the BMTL professional development workshop are as follows:

- Children will learn when they are fully engaged in mathematics;
- Children will learn what they observe through demonstrations that help them learn the structure of mathematics;
- Children will learn mathematics when they are immersed in mathematically-rich environments;
- Children will learn that they will and can use mathematics as they internalize the expectations from those they trust;
- Children will learn when they assume responsibility to choose when and how they will engage with mathematics;
- Children will learn when they are encouraged to use mathematics before it is fully mature and know that their developing thinking will be acknowledged and respected;
- Children will learn when they use mathematics in both social and solitary settings;
- Children will learn when they receive feedback on their mathematical thinking from those they trust during the entire learning process (Hopkins, 2007).

These conditions for learning are not naturally apparent or easily understood by early childhood mathematics teachers, and it was through the structured professional development experience that our participants came to view and understand how each of these components are essential to creating learning environments in which students can be successful in mathematics.

The Better Mathematics through Literacy (BMTL) project is a two-stage professional development program for K – 3 teachers and intervention specialists in a small section of Appalachia. In the first stage, during each of the week-long Summer Institute sessions, the teacher-participants are immersed and engaged in a mathematics learning community to explore student-centered mathematics instruction by the incorporation rich problems that create a classroom atmosphere conducive to meaningful learning, naturalistic inquiry, and the literary devices of writing, reading, and communicating. In the second stage that flows throughout the academic year, three follow-up sessions are held to provide opportunities to discuss further implementation of the program, to share further examples of mathematics instruction infused with literacy, and to build success for the program. As a culminating event, a final, conference-style Action Research Symposium is held in which each teacher-participant presents an encapsulated account of their project and the impact of the project on student learning.

Methodology

From a holistic research perspective quantitative and qualitative data were collected from the teacher participants in a pre/post structure during the year-long BMTL professional development experience. These measures included instruments to ascertain teachers' beliefs and dispositions about mathematics instruction, early childhood teachers' content knowledge in mathematics, and their evolving pedagogical approach to mathematics throughout the academic year. Additional instruments were created and piloted to solicit information about how the professional development impacted students' mathematical thinking and learning in the early childhood grades. A key piece of evidence that was used to inform the project team on the effectiveness of the BMTL approach was the BMTL Action Research Protocol, included as Appendix. To systematically and purposefully support the implementation of the student-centered approach to mathematics, the design team felt it was critical to track teachers' struggles and successes as they experimented with the BMTL strategies in their classrooms with young children. This protocol was an attempt on behalf of the design team to provide prolonged and persistent engagement with our participants while giving a tangible structure to the development of action research projects that would showcase the benefit of pedagogical approaches that integrate and infuse the seemingly separate worlds of literacy and mathematics.

Over the past four years of implementation, the BMTL project has reached nearly 200 early childhood classroom teachers and intervention specialists with approximately fifty teachers successfully completing the project each year. The classroom experience of our teacher-participants range greatly from induction year teachers to life-long classroom veterans with over twenty-five years working with young children. From the wide range of responses to the questions in the Action Research Protocol, two researchers purposefully selected a subset of twelve teacher-participant responses for in-depth analysis. The teachers in this sample represented an equal distribution across the early childhood grade levels and were also equally distributed across the four professional development cohorts. All of the selected teacher-participants were female, and the average classroom experience of the sample was thirteen years. The majority of the sample taught in self-contained classrooms throughout a small section of Appalachia with the exception of two intervention specialists who work with small groups of students with special needs in a resource room.

The Action Research Protocol was given to the BMTL teacher-participants during the last day of the intensive one-week Summer Institute as a way to support the purposeful and reflective examination of the changes the pedagogical approaches the participants were making in their classrooms as a result of their learning and experience with student-centered mathematics. Separate measures were used to gain an understanding of how these pedagogical changes impacted student learning in mathematics.

The responses to the Action Research Protocol questions were compiled in three pieces during the academic year follow-up sessions; the first was collected in late September, the second in early December, and the final installment in late February. Using an ongoing, recursive, and emergent approach (Merriam, 1988), the researchers read and coded the Protocol responses from the sample separately by looking for comments, statements, and evidence of pedagogical changes within the participants' classrooms and the impact of these changes on students' mathematical thinking and learning. These data excerpts were condensed into a chart to facilitate a cross-case analysis and to elicit emergent themes of pedagogical changes. The researchers met weekly to discuss and to confirm the similarities and differences in the data coding and iterative analysis. The major themes and findings from the sample yield evidence that the teacher-participants were able to make some progress toward a re-conceptualized and integrated pedagogical approach to mathematics that is consistent with the major tenets of mathematical reform.

Findings

In the examination of the sample teacher-participants' responses to the Action Research Protocol questions, the researcher team was able to trace how the classroom teachers and intervention specialists internalized the professional development from the Summer Institute, and how the BMTL approach was structured and implemented within their individual classrooms. All of the teachers and intervention specialists articulated detailed evidence of student work and personal, reflective narratives of how their daily mathematics instructional time became less textbook-driven as a result of their new, investigative approach to mathematics. Abstracting from the predominant themes identified, the teacher-participants became more integrated with literacy and language in the presentation of mathematical ideas and concepts, became more contextual in the tasks that they asked students to complete, and became more constructivist in their approach to teaching and learning mathematics.

Across the entire data sets that followed the teacher-participants across the academic year, the researchers described a natural, reflective analysis of the fragmented structure of the school day in early childhood. One teacher summarized, "We expected the children to use writing and language during the first two hours of the school day when we were in our Language Block, but after about a week of trying the BMTL strategies from this summer, I wondered why I never saw our math time in the same way" (Mrs. W., second grade teacher, Fall response, Question two). The project team felt that this response was representative of a pedagogical shift most teachers articulated as a result of their engagement and experiences in the BMTL professional development. By setting the expectation for using writing, speaking, and communicating to articulate students' developing mathematical thinking, the participants began to view their instruction as an opportunity for students to engage in making sense of the mathematics they were learning rather than memorizing a set of steps to carry out a procedure. The communication of thinking came to be viewed as an integral window into student thinking and understanding. As the classroom teachers became more secure in their student-centered approach to mathematics, the pressure to artificially 'cover' more content was subverted in order to provide students more authentic problem-solving.

A second emergent theme was that the teacher-participants articulated a more contextual approach to the tasks and situations that quickly became the basis of structured mathematics time with students. "I began using the characters and plot points from our literature series' Book of the Week as a context for leading students to think about addition and subtraction. It didn't take very long for the students to begin to make connections between the two and to begin to work together to use one [operation] to solve the other" (Ms. K., first grade teacher, Winter response, Question one). We felt this passage was indicative of classroom teachers adopting the BMTL approach in which mathematics lessons were fueled by the meaningful tasks and the use of collaboration as a tool to clarify and validate mathematical thinking. Concepts central to building mathematical understanding were no longer artificially separated into chapters or workbook pages devoid of any relevant context. Rather, elements of children's literature was used within the minds of children as a conduit to understanding as they actively and excitedly used a wide range of solution strategies to make sense of addition and subtraction in a context.

Finally, as a result of having a first-hand learning experience in which mathematics concepts were presented in a student-centered and constructivist framework as part of the Summer Institute, the BMTL participants were more comfortable implementing a similar instructional style in their own classrooms. Noticing that "the level of engagement is higher than I have ever seen during all of my teaching experience" (Ms. B, second grade teacher, Winter response, Question three) gave evidentiary weight to the concept that early childhood students could come to make sense of mathematics without being told what steps to follow. This, in turn, led the participants to provide more instructional time to the classroom conversation surrounding the development and articulation of mathematical thinking as more students wanted to share their ideas and make their solution strategies public.

Conclusion

The national conversation surrounding early childhood mathematics centers on how to strengthen classroom teachers' mathematical content knowledge and how to implement pedagogical changes that mirror the integrated, contextual, and constructivist approach supported by reform documents. Our research team feels that opportunities such as the Better Mathematics through Literacy professional development project can be meaningful to early childhood teachers and intervention specialists. By scaffolding the re-presentation of the mathematics they teach in student-centered and integrated ways, the participants are more willing to implement similar strategies in their own classrooms.

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APPENDIX
Better Mathematics Through Literacy (BMTL) Action Research Protocol

Monthly Planning Document for 2010-2011

August: Think about what you've learned in the intensive July workshop. Figure out what BMTL strategies (ways of teaching) you will integrate into your curriculum in 2010-2011
September: Be deliberate about what BMTL strategies (ways of teaching) you are using by keeping a journal. Besides being mindful to align your ways of teaching with Standards, be deliberate in examining the effect of your teaching (with BMTL) on student learning. The effect on student learning needs to be a continued and deliberate focus. The following questions may help structure your thinking in this regard:

1. How am I teaching? (i.e. What strategies am I using?)
2. What effect is the way I am teaching having on student learning?
3. How do I know that the way I am teaching is working (or not working) to improve student learning?
4. What sources of evidence will support the fact that the way I am teaching is having a positive effect on student learning? (Possible sources of evidence: student work, observations recorded in a journal, various forms of assessment, video tape or interview with students)

Fall Follow-up Session: Bring answers to the above questions (preferably word processed). We will spend some time debriefing on what's happening in your classrooms and how BMTL strategies (ways of teaching) are impacting student learning. *Bring two copies of your written answers—one for yourself and one for us to keep.*

October-November: Consider our discussion from the first follow-up session - what you heard from others about what is and isn't working. Utilize feedback from others and continue to be deliberate about how the way you are teaching relates to what and how your students are learning. Because we will be moving through an actual school year you will be utilizing more strategies or ways of teaching (and repeating some strategies) as the year goes on. Keep track of what strategies (ways of teaching) you are adding and how the strategies you are repeating over time impact student learning. Besides the original four questions (above) the following questions should help structure your thinking and move toward the Action Research Project:

1. What ways of teaching (strategies) have I used over a prolonged period of time?
2. What difference do I see in my students' learning now that they have more practice with these strategies and ways of thinking and learning?
3. What evidence do I have to support my conclusions in #2? (Here again, samples of student work, observations recorded in a journal, formal and informal assessments, video tapes of students working, and interviews with students would be excellent sources of evidence).

December Follow-up Session: Bring your answers to the above questions **<u>and some examples of student work</u>** that will show some of what's going on in your classroom as a result of BMTL. We will take time to share and generate feedback. *Bring two copies of your written answers—one for yourself and one for us to keep.*

January-February: Continue the process of being deliberate about your teaching and your students' learning as you employ strategies (ways of teaching) from BMTL. Because each follow-up session will present new information you should especially be mindful of strategies you are adding. For strategies you are continuing throughout the school year (for instance, if your students are keeping a math journal), your observations and supporting evidence of the effect on student learning over time are valuable. So besides the prior seven questions, you may want to ask the following:

1. Have I seen my students become more confident, comfortable, and capable with math because of the way I am teaching? Explain with some specific details which combine observation and supporting evidence.
2. Now that I'm 6 months into the school year and within three months of the Final Symposium for BMTL, what would I like to focus on in more depth? (i.e. What do I want to be the focus of my Action Research Project?)

Winter Follow-up Session: Bring answers to the above questions (optional) and the four questions listed below (required). *Bring two copies of your written answers—one for yourself and one for us to keep.* This is our last follow-up before the Final Symposium so you'll need to have a clear sense of direction on the specific aspect of BMTL and its effect on student learning that will be the topic of your Action Research Project. What we are looking for are the following:

- A clearly defined topic (a particular strategy or way of teaching) employed as a result of BMTL
- Conclusions about how the strategy/way of teaching affected student learning
- Evidence that supports your conclusions

The following questions will give shape to your Action Research Project:

1. What strategy (way of teaching) did I employ, and how was I deliberate in exploring the effects of this strategy or way of teaching on student learning? *You don't have to cover every strategy; focus on a particular strategy (way of teaching) or manageable combination of strategies.*
2. What was the effect of this strategy or way of teaching on student learning?
3. How do I know that this strategy or way of teaching impacted student learning in a given way? What evidence do I have to support my conclusions?
4. How can I share this research with others? (trifold, PowerPoint, essay of strategies and findings, video of students working, interviews with students, samples of student work, etc.)

AN ANALYSIS OF THINKING SKILLS FOR ALGEBRA I EOC TEST ITEMS

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This research provides insight into North Carolina's effort to incorporate higher-order thinking on its Algebra I End-of-Course tests. To facilitate the inclusion of higher-order thinking, the state used Dimensions of Thinking (Marzano, et al, 1988). An analysis of Algebra I test items found that the state's initial interpretation and application of Dimensions of Thinking was faulty and inconsistent; as a result, few Algebra I test items from 1998 and 2003 were found to assess higher-order thinking. Algebra I test items written in 2007 were found to be more cognitively complex and consistent with Dimensions of Thinking.

The ability to think at higher levels is generally considered a major instructional goal of education (Costa, 2001). However, many teachers find teaching for higher-order thinking (HOT) difficult, and some educators are concerned that the increase in state testing in the United States makes teaching for HOT even more challenging (Kohn, 2000; Ravitch, 2010). Their argument is that many state exams focus on lower order thinking (LOT) (i.e., procedural skills; symbol manipulation) at the expense of HOT (e.g., problem solving; reasoning). However, some propose that if state tests focused on HOT, then it might encourage and assist teachers to teach for HOT in their classrooms (Yeh, 2001).

State exams are political in nature since they help define what content is important, thus influencing how teachers teach and what students learn (McMillan, Myran, & Workman, 1999). Therefore, it is important to take a critical look at state testing programs. Since the 1990s, the North Carolina Department of Public Instruction (NC DPI) has used the core thinking skills from *Dimensions of Thinking* (Marzano et al, 1988) as a framework to develop test items for its End-of-Course examinations (NCDPI, 1999). The NC DPI (1996) chose to classify test items using *Dimensions of Thinking* because it allowed them to better assess

> … the mastery of higher level skills. The term "higher level skills" refers to the thinking and problem solving strategies that enable people to access, sort, and digest enormous amounts of information. It refers to the skills required to solve complex problems and to make informed choices and decisions. (p. 1)

The NC DPI made two modifications to *Dimensions of Thinking*. First, *Dimensions of Thinking* had eight categories of thinking: focusing, information gathering, remembering, organizing, analyzing, generating, integrating, evaluating. The NCDPI collapsed the first three categories – focusing, information gathering, and remembering – into a category called "knowledge". Second, the NC DPI added a thinking skill called "applying" not found previously in *Dimensions of Thinking*. According to NC DPI officials, "applying" is defined to be consistent with *Bloom's Taxonomy* (Bloom et al, 1956). They also explained that the thinking skills of knowledge, organizing and applying are considered LOT while analyzing, generating, integrating, and evaluating are considered HOT.

The research questions guiding this study were:

(1) How did NC DPI use *Dimensions of Thinking* to categorize thinking skills in its Algebra I End-of-Course tests between 1998 and 2007?

(2) Did NC DPI's use of *Dimensions of Thinking* result in the creation of Algebra I End-of-Course test items that were likely to assess higher-order thinking?

Defining higher-order thinking

As noted by Resnick (1987), thinking skills resist precise forms of definition; however, she believes that LOT and HOT can be recognized when each occurs. LOT is often characterized by the recall of information or the application of concepts or knowledge to familiar situations and contexts. Schmalz (1973) noted that LOT tasks require a student "… to recall a fact, perform a simple operation, or solve a familiar type of problem; it does not require the student to work outside the familiar" (p. 619). Senk, Beckman, & Thompson (1997) characterized LOT as solving tasks where the solution required applying a well-known algorithm. In general, LOT is generally characterized in the literature as solving tasks while working in familiar situations or contexts; or, applying well-known algorithms familiar to the students.

In contrast, Resnick (1987) characterized HOT as "non-algorithmic." Similarly, Stein and Lane (1996) described HOT as "the use of complex, non-algorithmic thinking to solve a task in which there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instruction, or a worked out example." (p. 58) Senk, et al (1997) characterized HOT as solving tasks where no algorithm has been taught. In general, HOT involves solving tasks where an algorithm has not been taught or using known algorithms while working in unfamiliar contexts or situations.

The main differences between LOT and HOT as described in the literature is the familiarity a student has to a task and whether the student already knows an algorithm or solution strategy that, when used properly, will lead him/her to the correct solution. For this research, the definition of Stein and Lane (1996) was used since it is specific to mathematics and best encapsulates most descriptions of HOT found in the literature.

Methods

Many of the test items and the thinking skills each measured needed for this research were available to the public on the NC DPI website. Seventy test items from 2003 and 72 test items from 2007 were downloaded for analysis. For 1998, NC DPI provided two Algebra I End-of-Course exams (Forms S and T) which had been released to the public, but were not available on-line; each 1998 EOC exam consisted of 81 items for a total of 304 Algebra I EOC test items.

For inter-rater reliability, a colleague in Educational Psychology and a former mathematics teacher also classified test items. To assist in categorizing test items the following materials were used as references:

- The modified *Dimensions of Thinking* framework materials published by NC DPI (1999)
- *Dimensions of Thinking* (Marzano et al, 1988)
- *Taxonomy of educational objectives: Part I, cognitive domain* (Bloom et al, 1956)
- *Handbook on formative and summative evaluation of student learning* (Bloom, Hasting, & Madaus, 1971)

Bloom's materials were used since the NC DPI framework included "applying" from *Bloom's Taxonomy*.

In March 2010, the Algebra I test items from 1998, 2003, and 2007 were categorized using the definition of LOT / HOT as defined by Stein and Lane (1996). The inter-rater reliability for the 304 test items was 89%. According to Kaid and Wadsworth (1989), researchers can generally be satisfied with inter-reliability greater than 85%. Items for which the raters disagreed were resolved in NC DPI's favor. In the summer of 2010, all 304 Algebra I EOC test items from 1998 – 2007 were organized by thinking skill as classified by NC DPI. Similarities and differences within the classifications were identified to ascertain how the NC DPI applied *Dimensions of Thinking* for these test items. NC DPI's categorization of thinking skills were analyzed using descriptive statistics.

Results

The classification of Algebra I EOC test items varied over the years. Table 1 summarizes the number (percent) of thinking skills used for test items.

Table 1.

NCDPI classification of algebra I test items, 1998 – 2007

NC DPI Classification	**1998 # (%)**	**2003 # (%)**	**2007 # (%)**
Knowledge	10 (6%)	2 (3%)	0 (0%)
Organizing	2 (1%)	4 (6%)	5 (7%)
Applying	88 (54%)	26 (37%)	40 (56%)
Analyzing	46 (28%)	24 (34%)	27 (38%)
Generating	5 (3%)	1 (1%)	0 (0%)
Integrating	9 (6%)	13 (19%)	0 (0%)
Evaluating	2 (1%)	0 (0%)	0 (0%)
Totals	**162 (99%)***	**70 (100%)**	**72 (101%)***

* Percent ≠ 100% due to rounding

Between 1998 and 2007, a notable difference is observed regarding how often thinking skills were used to classify test items. In 1998, all seven thinking skills were used at least twice; however, by 2007, only the three thinking skills of organizing, applying and analyzing were used. According to an NC DPI official, this was done to make the process of classifying test items more coherent and consistent without losing the integrity of incorporating HOT in the End-of-Course exams; in this case, the thinking skills "organizing" and "applying" represented LOT and the thinking skill "analyzing" represented HOT.

In the analysis of the Algebra I EOC test items, three patterns emerged:

(1) It was not unusual for the same (or almost identical) test items to be classified using different thinking skills

(2) A large number of test items did not match their description in *Dimensions of Thinking* or *Bloom's Taxonomy*

(3) The findings of (1) and (2) were restricted to 1998 and 2003; by 2007, the percent of HOT algebra I test items increased significantly and appeared appropriately classified.

Inconsistent identification of thinking skills

Test item classification was inconsistently used by the NC DPI. Examples of inconsistent identification of a thinking skill are included in Table 2. All test items were multiple choice; however, the choices are not included in the table.

Table 2.

Examples of items classified using more than one thinking skill

***NC DPI* Classifications**	**Test Item**
Knowledge & Analyzing	Which is the graph of a line with a slope of 2 and a y-intercept of 5?
Applying & Integrating	What is the sales tax on a $15,000 boat if the sales tax rate is 6%?
Applying & Analyzing	Solve: $-4 \leq 3x + 2 \leq 5$
Applying & Analyzing	What is $4x + 5y + 9 = 0$ in slope-intercept form?
Applying &Analyzing	What is the greatest common factor of $15x^4y^7 – 21x^7y^7 + 6x^2y^2$?

Misclassification of thinking skills

It was not unusual for the thinking skill classifications of test items to not match their descriptions in *Dimensions of Thinking* or *Bloom's Taxonomy*. Examples of misclassification of applying, analyzing, generating, and integrating are provided below.

Applying

In *Bloom's Taxonomy*, for a test item to be at the level of application or higher, a "new situation" for the student is required. Bloom emphasized in his original work in 1956 and subsequent discussions on this issue (e.g., Bloom et al, 1971) that application and higher levels in the taxonomy do not refer to test items where only minor changes are made, but otherwise, the procedure was the same to that practiced in class. Bloom stated that similar tasks that had already been practiced in class would be labeled "comprehension" (Bloom, et al, 1971). The following table includes examples of test items considered to be mislabeled by NC DPI as applying.

Table 3.

Examples of test items considered to be mislabeled as applying

Simplify: $\lvert -4 + 2 \rvert$	Factor: $6x^2 – 23x + 10$
Simplify: $(3b^2c)(8b^3c^6)$	Solve: $(x – 20)^2 = 100$
Simplify: $(5x + 2) + (2x + 8)$	Solve: $x + 8 > 7$ $x \in \{-3, -1, 0, 1, 3\}$
Solve: $(3x + 6)(2x – 1) = 0$	Solve: $6x – 12x – 4 = 68$

These test items are likely to be routine and familiar to students after taking Algebra I; thus would not be considered application in *Bloom's Taxonomy* (and therefore, by default, at the knowledge or organizing levels within *Dimensions of Thinking*). However, the type of items labeled applying changed between 1998 and 2007. In 1998, 88 out of 162 (54%) test items were labeled as applying but only two of these items were placed in a real world context. Of the 20 test items in 1998 that were placed in context, almost all were labeled analyzing or integrating (HOT). However, by 2007, 40 out of 72 (56%) test items were labeled application; of these, 21 were word problems placed in a real world context. Although labeled as applying in 2007, most of these types of test items would have been labeled as analyzing or higher in 1998. For example, the following item was labeled applying (LOT) in 2007 while an almost identical test item was labeled analyzing (HOT) in 1998.

An object is blasted upward at an initial velocity, v_o, of 240 ft/s. The height, h(t), of the object is a function of time, t (in seconds), and is given by the formula $h(t) = v_o t - 16t^2$. How long will it take the object to hit the ground after takeoff?

Despite NC DPI's goal to use *Dimensions of Thinking* as the framework to incorporate HOT in its EOC exams, 51% of test items between 1998 – 2007 were classified as "applying," (a non- *Dimensions of Thinking* category). Thus, *Dimensions of Thinking* was not used most of the time to classify test items.

One of the characteristics of a HOT test item is its "newness" to the solver or its non-routine nature. However many of the items labeled as HOT by NC DPI were procedural and routine (i.e., LOT). This was mostly prevalent in 1998 and 2003. Examples of the misidentification of "HOT" test items are included below.

Analyzing

- *The formula to find the area of a circle is $A = \pi r^2$. What is the area of a circle if the diameter is 16 cm? (Use 3.14 for π)*
- *What is the greatest common factor of $-2r^6st^2 - 2r^3s^3t + 16r^2s^3t$?*
- *Which of the following is an algebraic expression for "twice the sum of a number and 5"?*

Generating

- *The sum of a number and ten is twelve." What is the equation for this statement?*
- *There are 24 yards of rope with which to enclose a rectangular area. If* w *is the width of the rectangle, what is the area function for the roped-off rectangle?*

Integrating

- *The length of a house is 68 feet and the width is 24 feet. Find the area of the house.*
- *What is the sales tax on $10,200 automobile if the sales tax rate is 4%?*

In some of the examples above, it appears that test items were classified using common mathematical and pedagogical definitions of these terms in place of their meaning in *Dimensions of Thinking*; for example, *generating* an equation or *integrating* subject matter. Research indicates that misapplication of thinking skills in mathematics is not uncommon (Gierl, 1997).

Increase in HOT test items over time

Algebra I test items misidentified as HOT were significantly more prevalent in 1998 and 2003. By 2007, many of the test items labeled analyzing (the only HOT thinking skill used in 2007) appeared to be appropriately classified. In general, the 2007 test items were much more complex and put greater emphasis on problem solving and conceptual understanding. In 1998, NC DPI classified 37% of test items as HOT while only 14% were classified as HOT in this study. However, by 2007, NC DPI classification of test items as HOT mirrored the researcher classifications. Table 4 compares the NC DPI and researcher classifications.

Table 4.

Classification of test items as HOT vs. LOT, 1998 – 2007

	1998		**2003**		**2007**	
	NC	**R**	**NC**	**R**	**NC**	**R**
LOT	61%	86%	46%	76%	63%	64%
HOT	38%	14%	54%	24%	38%	36%

NC = NCDPI; **R** = Researcher * Percent ≠ 100% due to rounding

Discussion

In the initial years of using *Dimensions of Thinking*, the thinking skills used to label test items were often not consistent with how they are defined in *Dimensions of Thinking*. In 1998 and 2003, the majority of Algebra I EOC test items classified as HOT by NC DPI were routine mathematics exercises for which students were very likely to have been taught an algorithm or procedure to solve. In 1998 and 2003, a key concept in the literature on HOT and in *Bloom's Taxonomy* that appeared to be missing from NC DPI's interpretation of HOT was the level of *familiarity* students had with the algorithms or methods of completing a task. As a result, NC

DPI initially misinterpreted *Dimensions of Thinking* and over-estimated the amount of HOT on its Algebra I EOC exams. This appeared to have been corrected by 2007 where items labeled as LOT / HOT were consistent with the literature on HOT and the definitions of LOT / HOT used for this research.

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INDIVIDUAL DIFFERENCES IN MENTAL REPRESENTATIONS OF FRACTION MAGNITUDE

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The numerical distance effect is a robust effect in mathematical cognition that describes a negative correlation of the numerical distance between two numbers and the time it takes to choose the larger number. The presence of this effect is commonly taken as evidence for a person's tendency to represent numbers conceptually on a mental number line; i.e, a magnitude-based representation.. In the current study, the size of the numerical distance effect decreased for individuals with high mathematics anxiety or high calculator use, indicating that those individuals tend to have less-developed magnitude-based representations of fractions.

Mathematical tasks induce people to form a wide range of mental representations of number. For example, when people are asked to quickly choose the larger of two numbers, they do so more quickly and accurately when the distance between the numbers is relatively large, compared to when the distance between the two numbers is small (e.g, Moyer & Landauer, 1967; Dehaene, 1992). Moreover, the response times tend to decrease as either a logarithmic function (Dehaene, Dupoux, & Mehler, 1990) or a linear function (Gallistel & Gelman, 2000) of the increasing distance between the two numbers. This is a robust effect in mathematical cognition known as the *numerical distance effect*, and its presence suggests that people use an analog magnitude-based representation (e.g., a mental number line) to compare natural numbers.

Recently, several researchers have begun to investigate the mental representations that people use when thinking about fractions. Bonato, Fabbri, Umiltà, and Zorzi (2007) had participants press a button to choose the larger of two fractions presented on a computer screen. They found that participants tended to compare the components of the fractions (numerators and denominators) and not the real numerical value (or magnitude) of the fractions. This led them to conclude that people did not form mental representations of fraction magnitude. In contrast, both Meert, Grégoire, and Noël (2009) and Schneider and Siegler (2010) found significant numerical distance effects in fraction comparison tasks, indicating that people do indeed form magnitude-based mental representations of fractions. Similarly, Faulkenberry and Pierce (2010) found that people exhibit a significant numerical distance effect regardless of the type of strategy (conceptual or procedural) employed to compare fractions, again indicating the presence of a magnitude-based fraction representation. However, the size of the distance effect (as measured by the coefficient of determination r^2) varied across the types of strategies used.

The current study investigated the influence of individual differences on magnitude-based representations of fractions. Affective variables are known to have significant effects on various aspects of mathematical cognition. Of particular interest to the current study are math anxiety (Ashcraft & Kirk, 2001), arithmetic skill (Campbell & Xue, 2001; LeFevre & Bisanz, 1986), and daily calculator use (Imbo & Vandierendonck, 2007; but see Campbell & Xue, 2001). All of the above-mentioned affective variables tend to negatively affect performance as measured by RT or error rates (or both). Given this, it is possible that having a detrimental level of one of these variables would result in a less-pronounced numerical distance effect when comparing fractions. That is, it is possible that these variables could be negatively associated with the successful use of the mental number line.

Participants were asked to make speeded judgments of fraction magnitude for simple proper fractions. For each participant, a measure of the size of the numerical distance effect was computed by regressing reaction time against the numerical distance between the two fractions and computing the coefficient of determination (r^2) for that relationship. The higher the value for r^2, the larger the numerical distance effect. Specifically, it was predicted that individuals who reported high levels of math anxiety or high amounts of calculator use would exhibit a smaller numerical distance effect, compared to those individuals who reported low levels of math anxiety or low amounts of calculator use. Also, it was predicted that individuals with higher arithmetic fluency would exhibit a larger numerical distance effect than those individuals with lower arithmetic fluency.

Method

Participants

Twenty-eight undergraduate students (21 female) from Texas A&M University – Commerce participated in the current study. The participants were volunteers from several freshman-level mathematics courses who took part in the study for partial course credit. The mean age was 27.3 years (range 18-55 years; median 25 years, standard deviation 8.41 years.

Experimental Stimuli and Measures

The set of fraction stimuli was set of 48 reduced, proper fraction pairs that consisted of three sets of 16 fractions. Each of the sets of 16 contained one of three critical fractions for comparison: 1/2, 1/3, or 2/3. In each group of 16, half of the fractions were less than the given critical comparison fraction, with the other half greater. Also, the left-side/right-side status of the larger fraction was equally distributed within each group of 16. No fraction pairs were repeated, but each individual fraction was presented twice (in different left/right positions) in comparison with two different critical fractions.

In addition, each participant completed a demographic survey asking for subjective ratings (on an integer scale of 1=low to 5=high) of their level of mathematics anxiety and tendency to use a calculator for routine computations. Each participant was also assessed on their arithmetic fluency by completing the Addition test and the Subtraction-Multiplication test from the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Dermen, 1976). The Addition test was composed of two pages of three-addend addition problems (for a total of 120 problems). The Subtraction-Multiplication test consisted of two pages of two-digit subtraction problems and two-by-one digit multiplication problems (for a total of 120 problems). Participants were allowed 2 minutes per page to correctly answer as many problems as they could. Arithmetic fluency was defined as the total number of correct answers on both tests.

Procedure

Participants were first given an instruction phase that consisted of three simple fraction comparisons: 1/2 vs. 1/3, 1/2 vs. 2/3, and 1/3 vs. 2/3. Participants were told to answer as quickly and accurately as possible. Feedback was presented in the form of an audible beep (for correct answers) and an audible buzz (for incorrect answers). Once the instruction phase was complete, participants were given a chance to ask any questions of the experimenter before the testing phase began.

During the testing phase, no feedback was given. Each trial began with the sentence, “Say ‘Go’ when ready,” presented in the center of the screen. Through a lapel microphone, the participant’s vocalization triggered the software to present a fraction pair, which remained on the screen until a button was pressed, the side indicating which fraction was larger in magnitude, or 15 seconds elapsed.

Results

A total of 1344 trials were administered. Of these trials, 30 trials were discarded due to either a failure in the experimental apparatus or a failure to respond within 15 seconds. Of the remaining 1314 trials, 177 were answered incorrectly, resulting in an overall error rate of 13.5%.

The median solution time across these remaining trials (including trials on which an error was committed) was 3142 ms, SD = 3311 ms.

At the item level, a regression analysis using the distance between the numerical values of fraction pairs as a predictor of median reaction time across all participants and trials showed a significant numerical distance effect, with numerical distance accounting for 44% of the variance in median reaction times ($t(46) = -6.06$, $p<0.001$). Remarkably, a regression analysis using the natural logarithm of the numerical distance between fractions as a predictor of median reaction time exhibited a numerical distance effect of almost equal size, with the logarithm of numerical distance accounting for 43% of the variance in median reaction times ($t(46)=-5.96$, $p<0.001$). Figure 1 shows side-by-side scatter plots representing both models. Since the logarithmic model accounted for no more variance in median reaction time than the linear model, no further consideration of the logarithmic model was made.

At the participant level, linear regression analyses predicting RT as a function of numerical distance showed that 18 of the 28 participants exhibited a significant numerical distance effect. For those participants with a significant numerical distance effect, numerical distance between fractions accounted for an average of 23.0% of the variance in reaction times (standard deviation = 8.3%). For participants who did not exhibit a significant numerical distance effect, numerical distance between fractions only accounted for 2.4% of the variance in reaction times (standard deviation 2.7%).

Further analysis of the contribution of individual differences to the size of the numerical distance effect showed marked differences (see Figure 2). To analyze the effect of mathematics anxiety, participants were grouped according to their subjective rating (1=low to 5=high) on the mathematics anxiety question in the demographic survey. Those participants rating themselves with a 1 or a 2 were classified as having "Low" mathematics anxiety, and those participants rating themselves as 4 or 5 were classified as having "High" mathematics anxiety. Four

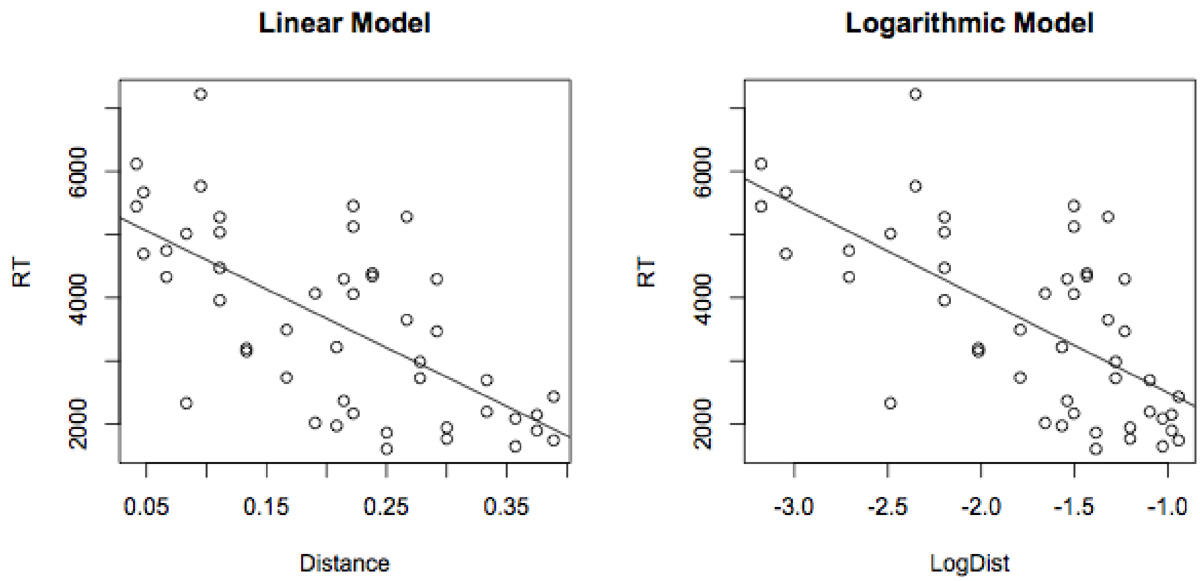

Figure 1. Scatter plots of median reaction time versus numerical distance between fractions. Both the linear and logarithmic models predict an equal amount of variance in the median reaction times.

participants rated themselves as 3, and were excluded from this analysis. Participants classified as Low Mathematics Anxiety exhibited a much greater numerical distance effect than those participants classified as High Mathematics Anxiety ($F(1,22) = 10.41$, $p=0.004$). That is, for participants with Low Mathematics Anxiety, numerical distance between fractions accounted for 23.1% of the variance in reaction time, whereas for participants with High Mathematics Anxiety, numerical distance only explained 9.2% of the variance in reaction times.

A similar analysis was conducted for daily calculator use. Participants were classified in a manner identical to the method for math anxiety. For reasons as above, five participants were excluded from this analysis. Participants classified as Low Calculator Use exhibited a much greater numerical distance effect than those classified as High Calculator Use. Numerical distance accounted for 26.2% of the variance in reaction time for those participants who rarely used calculators, compared to 9.9% for those who used calculators often.

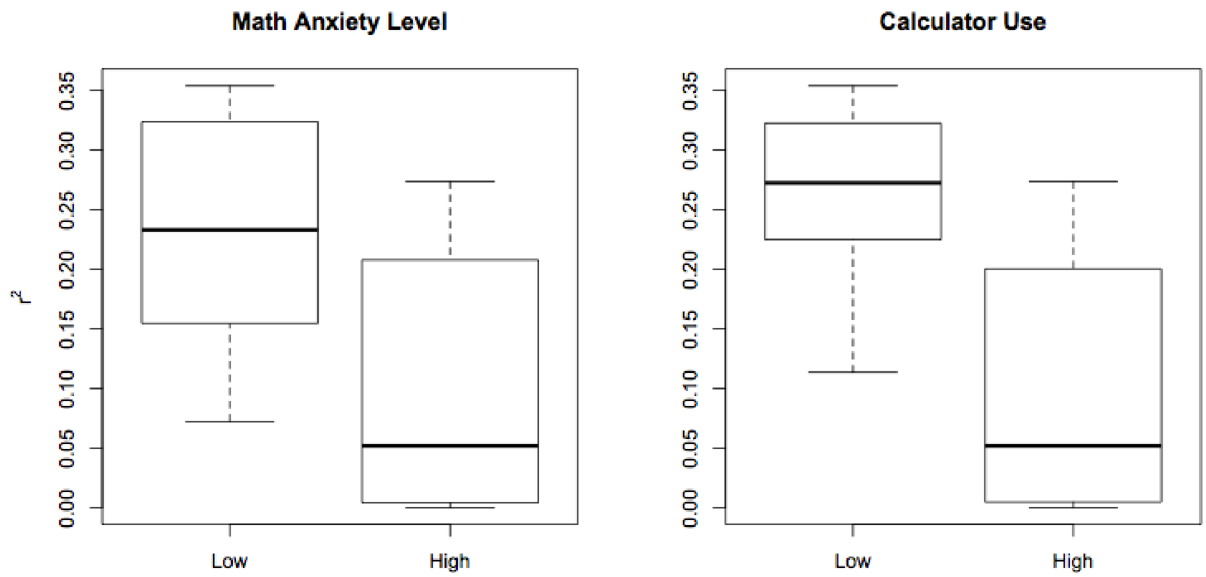

Figure 2. The size of the numerical distance effect as a function of individual differences in math anxiety level and daily calculator use.

Finally, the contribution of arithmetic fluency was analyzed by regressing an individual's coefficient of determination (r^2) with the score on the arithmetic fluency test as a predictor. This analysis showed virtually no effect of arithmetic fluency on the size of the numerical distance effect ($r^2 = 0.003, F(1,26)=0.08$).

Discussion

Participants tended to correctly select the larger of two presented fractions more quickly when the fractions presented were farther apart on the number line, compared to when the fractions were close together. This is typically thought to correspond to an integrated, magnitude-based representation that is akin to a mental number-line. This finding replicates the core finding of several recent studies (Faulkenberry & Pierce, 2010; Schneider & Siegler, 2010; Meert, Grégoire, & Noël, 2009) with one exception. The current study found that median RT is predicted best by a linear function of the numerical distance between fractions, whereas Schneider & Siegler (2010) showed that median RT was best predicted by the logarithm of the numerical distance. Nonetheless, the current data lends further support for the numerical distance effect as a robust effect in mathematical cognition.

The current study takes an additional step of considering individual differences as a predictor of the extent to which participants possess and use a well-developed mental number line for fractions. This extent was measured by the size of the numerical distance effect for each participant. Participants with a low level of mathematics anxiety tended to exhibit much larger numerical distance effects than those with a high level of mathematics anxiety. That is, individuals with low math anxiety tend to have more robust magnitude-based representations of fractions than their high math anxiety counterparts. Similarly, participants who use calculators very little in daily life were also found to have more robust magnitude-based representations of fraction than their counterparts who use calculators often. Perhaps surprisingly, arithmetic fluency had virtually no effect on a person's tendency to use a magnitude-based representation.

The use of regression parameters to quantify aspects of an individual's mental representation of number is not new (e.g., Salthouse & Coon, 1994; Geary, Frensch, and Wiley, 1993) and can illuminate many individual differences that would not be visible with raw reaction time data. Indeed, the individual regression parameters provide a way to standardize reaction time data that removes the influence of performance differences (such as prior knowledge and practice effects) and instead relies on within-subject patterns of performance.

In summary, the current study found that individuals with a high level of mathematics anxiety or a high propensity for calculator use tend to rely less on magnitude-based mental representations of fractions. Future research should attempt to study the consequences of these representational shifts, especially with respect to individuals' procedural and conceptual knowledge of fractions.

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A REVISON OF THE MATHEMATICS TEACHING EFFICACY BELIEFS INSTRUMENT FOR KOREAN PRESERVICE TEACHERS

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This study sought to revise the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) for Korean preservice teachers. The MTEBI was translated into Korean and four Korean mathematics teacher education professors were asked to analyze the instrument to ascertain if it would be appropriate for use within the Korean contexts of academic setting and culture. The professors concluded that 14 of the 21 items on the MTEBI were inappropriate. Their concerns included awkward wording, tense disagreement, vagueness, and multiple meanings. As a result, these items were modified to better fit the Korean language and culture.

A teacher's self-efficacy is a significant psychological construct that influences instructional performances and student outcomes (Gibson & Dembo, 1984). Self-efficacy emphasizes the extent to which teachers believe they control, or at least strongly influence, student achievement and motivation (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998). Teacher efficacy is refined as "a teacher's judgment of his or her capabilities to bring desired outcomes of student engagement and learning, even among those students who may be difficult or unmotivated" (Tschannen-Moran & Hoy, 2001, p783). For over 30 years, educational researchers have studied teacher efficacy, which includes how it can best be measured, how it is relates to other variables such as student achievement, and the significance of it.

The nature of teacher efficacy, however, may vary according to the academic discipline (Tschannen-Moran & Hoy, 2001) and from one culture to the next (Lin & Gorrell, 2001). Addressing these two issues, this article describes the authors' efforts to revise the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI)—developed in the US—for Korean elementary preservice teachers. The revisions were completed by having four Korean mathematics teacher education professors provide their perspectives on each item of the MTEBI. It is expected that a revised MTEBI adapted to the Korean culture will provide a more valid and reliable measure of Korean preservice teachers' mathematics teaching efficacy beliefs.

Related Research

Bandura's self-efficacy theory (1977, 1986, 1997) has been influential in the study of preservice teachers' efficacy beliefs. Bandura (1977) defined self-efficacy as individuals' judgments of their capabilities to accomplish certain levels of performance. According to Bandura, a person's future behavior can be better predicted through one's efficacy beliefs than through actual accomplishments (1986). He also presented a two-dimensional model of self-efficacy in which an individual's behavior is influenced by both personal efficacy and outcome expectancy. Personal efficacy is an individual's beliefs that influence one's capability to cope with change in situated experiences, and outcome expectancy is a generalized expectation that influences an individual's action-outcome contingencies based on perceived life experiences.

Taking this assertion into the context of teaching, Gibson and Dembo (1984) suggested studying teacher efficacy. Gibson and Dembo (1984) adapted Bandura's theory to develop the Teacher Efficacy Scale. This instrument consists of two subscales measuring Teaching Efficacy (TE)-it was later identified as Personal Teaching Efficacy (PTE) by other researchers—and General Teaching Efficacy (GTE). These two subscales correspond to Bandura's SE and OE, respectively. Gibson and Dembo's scale has been widely used in studies that verified the importance of teacher efficacy as a construct (Tschannen-Moran et al., 1998).

Teacher efficacy measures have been developed within the specific subject of mathematics. Enochs, Smith, and Huinker (2000) developed the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI), with the two scales, Personal Mathematics Teaching Efficacy (PMTE), corresponding to Bandura's SE and Gibson and Dembo's PTE, and Mathematics Teaching Outcome Expectancy (MTOE), corresponding to Bandura's OE and Gibson and Dembo's GTE. The MTEBI has been used in research to measure the teaching efficacy of US elementary preservice teachers (Gresham, 2008; Swars, 2005; Swars, Daane, & Giesen, 2006; Swars, Smith, Smith, & Hart, 2009; Utley, Bryant, & Moseley, 2005).

Since teacher efficacy may vary from one culture to the next (Lin & Gorrell, 2001), it is questionable whether the MTEBI, as it is currently written, can be used in other cultures. The MTEBI has been tested in a few non-Western cultures. For example, Alkhateeb (2004) translated the MTEBI into Arabic to verify its accuracy in Jordan. However, since English and Korean are very different languages, researchers should consider linguistic and socio-cultural dimensions when translating the instrument from English to Korean.

Method

Participants

Four Korean mathematics teacher education professors participated in this study. Each professor is either the department chair or program coordinator of mathematics education in a College/University of Education in South Korea. Two of them earned their doctoral degree in Korea; others earned in Canada and the US. One professor is female; others are male. Their professional careers ranged from 8 to 20 years.

Instrument

The MTEBI measures the degree of a preservice teacher's feeling that s/he teaches mathematics effectively. The MTEBI consists of two scales, the Personal Mathematics Teaching Efficacy (PMTE) with 13 items, and the Mathematics Teaching Outcome Expectancy (MTOE) with eight items. Each PMTE item is stated in the first person and written in the future tense since preservice teachers are not yet professional teachers; eight PMTE items are negatively worded. Each MTOE item is stated in the third person and written in the present tense; they are all positively worded. For convenience, a PMTE item was coded by the initial P with its item number, and an MTOE item was coded by the initial O with its item number (e.g., P2, O7)

Translation

One of the authors along with two bilingual doctoral students, who were knowledgeable of the concept of mathematics teaching efficacy, translated the MTEBI into Korean. In translating from one language to another, it is important to conduct a back-translation to check the translation quality (Brislin, 1970). The Korean-translated MTEBI was translated back into English by another bilingual graduate student. Then, comparing the original MTEBI, the translated MTEBI, and the back-translated MTEBI led to some modifications in the Korean version MTEBI.

Procedure

The four Korean mathematics education professors were asked to review the Korean-translated MTEBI. Interviews with the four reviewers were conducted through e-mails and a meeting at an international conference in Korea. The reviewers checked the translation and appropriate use of language of the MTEBI, especially the language used for mathematics teacher education in Korean classrooms. In the first e-mail, a description on the MTEBI was provided to the reviewers. The main question given to the reviewers was: *Do you believe that each item is* *appropriate for measuring a preservice teacher's mathematics teaching efficacy beliefs regarding their mathematical knowledge, skills, and behavior? Why or why not?*

After reviewing the MTEBI, each reviewer was asked to give additional comments or suggestions of the other reviews. Follow-up discussions between each reviewer and the authors were completed through e-mails. One of the authors also met two of the reviewers at an international conference to discuss the reviews in person. Additional follow-up discussions after the meeting in the conference through e-mail led to final agreement on the appropriateness of each item in measuring a preservice teacher's personal mathematics teaching efficacy and mathematics teaching outcome expectancy.

Results

All reviewers agreed an item to be appropriate when no changes were needed in the statement of the item. Reviewers considered three PMTE items (P8, P15, P16) and four MTOE items (O4, O7, O12, O13), to be appropriate (See Table 1).

Table 1.

Appropriate Items

Code	Item
O4	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.
O7	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.
P8	I will generally teach mathematics ineffectively.
O12	The teacher is generally responsible for the achievement of students in mathematics.
O13	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.
P15	I will find it difficult to use manipulatives to explain to students why mathematics works.
P16	I will typically be able to answer students' questions.

The reviewers agreed that the content of the items regarding mathematics teaching efficacy beliefs were acceptable. However, they indicated that some items were problematic once they were translated into Korean. These problems were classified as (a) awkward wording, (b) tense disagreement (c) vagueness, and (d) multiple meanings. Table 2 shows the original MTEBI items as written in English and the problems found by the reviewers after the items were translated into Korean.

Table 2.

Inappropriate Items and Problems in Its Korean Translation

Code	Item	Problems
O1	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	Multiple meaning
P2	I will continually find better ways to teach mathematics.	Vagueness
P3	Even if I try very hard, I will not teach mathematics as well as I will most subjects.	Awkward wording & Multiple meaning
P5	I know how to teach mathematics concepts effectively.	Tense
P6	I will not be very effective in monitoring mathematics activities.	Awkward wording
O9	The inadequacy of a student's mathematics background can be overcome by good teaching.	Vagueness
O10	When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.	Vagueness
P11	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	Tense
O14	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	Vagueness
P17	I wonder if I will have the necessary skills to teach mathematics.	Multiple meaning
P18	Given a choice, I will not invite the principal to evaluate my mathematics teaching.	Awkward wording & Multiple meaning
P19	When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	Awkward wording
P20	When teaching mathematics, I will usually welcome student questions.	Awkward wording
P21	I do not know what to do to turn students on to mathematics.	Awkward wording

An item is considered to be *awkward* when the language used is contrary to a usual way it is expressed in Korean. The reviewers discussed that "[It] is okay but not often used; is understandable but not easily acceptable; and is not going well with other parts," and "it is awkward." *Tense disagreement* occurred when a PMTE item was stated in the present tense instead of in the future tense. *Vagueness* was determined when an unclear word was used in the item. Reviewers stated, "[The word] is vague so the meaning of the statement is unclear." The problem of *multiple meanings* occurred when an item could be interpreted in more than one way. One of the interesting findings is that all six awkward items were PMTE items (P3, P6, P18, P19, P20, P21), and among them, 5 items (P3, P6, P18, P19, P21) were negatively worded.

Conclusion

In cross-cultural studies, equivalence between the source and target instrument should be seriously and carefully thought through. Data obtained from translated measures that have not been evaluated for cultural or language equivalence is meaningless (Sperber, Devellis, & Boehlecke, 1994). Since linguistic usage is considerably different across cultures, equivalence between two languages is unlikely to be attained by using a word-by-word translation. When an instrument is translated from one language to another, grammatical sensitivity as well as connotative characteristics including culture, experience, syntax, and conceptual interpretation need to be considered (Wang & Lee, 2006). The MTEBI, in this study, was carefully translated into Korean with this consideration.

This study found that once the MTEBI was translated into Korean, 14 out of 21 items were found to be problematic. Categories of problems included: awkward wording, tense disagreement, vagueness, and multiple meanings. By restating the problematic items, a revised Korean version MTEBI was developed. Once additional research is conducted to test for reliability and validity of the revised Korean MTEBI, it is anticipated that it will provide appropriate information regarding mathematics teaching efficacy among Korean elementary preservice teachers.

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TEACHER DIFFERENCES IN MATHEMATICS KNOWLEDGE, ATTITUDES, AND SELF-EFFICACY AMONG NYC TEACHING FELLOWS

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Providing students in urban settings with quality teachers is important for student achievement. This study examined the differences in content knowledge, attitudes toward mathematics, and self-efficacy among teachers in the NYC Teaching Fellows program, and informs teacher education in mathematics alternative certification. Teaching Fellows were given a mathematics content test and two questionnaires, and took a standardized test. Findings revealed that high school teachers had significantly higher content knowledge than middle school teachers. Mathematics Teaching Fellows had significantly higher content knowledge than Mathematics Immersion Teaching Fellows. Mathematics and science majors had significantly higher content knowledge than other majors.

The purpose of this study was to determine differences in content knowledge, attitudes toward mathematics, and concepts of teaching self-efficacy among different categories of alternative certification teachers in New York City. The teachers in this study come from two mathematics methods sections of New York City Teaching Fellows (NYCTF) teachers. The NYCTF program was developed in 2000 in conjunction with The New Teacher Project and the New York City Department of Education (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007; NYCTF, 2008). The program goal was to recruit professionals from other fields to supply the large teacher shortages in New York City's public schools.

Background and Theoretical Framework

Previous research found that teachers prepared in alternative certification programs, such as the Teaching Fellows program, have on average higher test content scores than other teachers (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2006; Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007). However, details about content knowledge have been sparse and there has been a lack of concentrated focus on mathematics teachers specifically. Most studies investigated teacher retention and student achievement as variables to determine success. These are two of the most important variables, but there is a need to investigate other variables related to success, such as teacher content knowledge, attitudes toward mathematics, and teacher self-efficacy. Humphrey and Wechsler (2007) called for more research into teachers' backgrounds in alternative certification pathways: "Clearly, much more needs to be known about alternative certification participants and programs and about how alternative certification can best prepare highly effective teachers" (p. 512).

Aiken (1970) and Ma and Kishor (1997) found a small but positive significant relationship between achievement and attitudes. This relationship between achievement and attitudes, along with Ball, Hill, and Bass' (2005) emphasis on the importance of content knowledge for teachers, formed the framework of this study. Additionally, Bandura's (1986) construct of self-efficacy theory framed the study's focus on self-efficacy. Bandura found that teacher self-efficacy can be subdivided into a teacher's belief in his or her ability to teach effectively, and his or her belief in affecting student learning outcomes. Teachers who feel that they cannot effectively teach mathematics and affect student learning are more likely to avoid teaching from an inquiry student-centered approach with real understanding (Swars, Daane, & Giesen, 2006).

Research Questions

1. Are there differences in mathematical content knowledge, attitudes toward mathematics, and concepts of teacher self-efficacy between middle and high school Teaching Fellows?
2. Are there differences in mathematical content knowledge, attitudes toward mathematics, and concepts of teacher self-efficacy between Mathematics and Mathematics Immersion Teaching Fellows?
3. Are there differences in mathematical content knowledge, attitudes toward mathematics, and concepts of teacher self-efficacy between undergraduate college majors among the Teaching Fellows?

Methodology

The sample in this quantitative study consisted of 42 new teachers in the Teaching Fellows program enrolled in a master's degree program from two sections of mathematics methods that involved a combination of both pedagogical and content instruction. The course focused on constructivist methods with an emphasis on problem solving and real-world connections. Teaching Fellows were labeled as Mathematics or Mathematics Immersion students based upon having 30 or more mathematics undergraduate content credits before entering the program. Mathematics Teaching Fellows have the required minimum 30 credits, while Mathematics Immersion Teaching Fellows do not.

Teaching Fellows were given a mathematics content test and two questionnaires at the beginning and end of the semester. The mathematics content test consisted of 25 free response items ranging from algebra to calculus. Additionally, mathematics Content Specialty Test (CST) scores for the New York State certification were recorded as another measure of mathematical content knowledge.

The first questionnaire was created by Tapia (1996) and has 40 items that measured attitudes toward mathematics including self-confidence, value, enjoyment, and motivation in mathematics using a 5-point Likert scale. The second questionnaire was adapted from the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) developed by Enochs, Smith, and Huinker (2000), and measured concepts of self-efficacy with 21-items using a 5-point Likert scale instrument. The MTEBI contains two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE) with 13 and 8 items, respectively. Possible scores range from 13 to 65 on the PMTE, and 8 to 40 on the MTOE. The PMTE specifically measures a teacher's self-concept of his or her ability to effectively teach mathematics. The MTOE specifically measures a teacher's belief in his or her ability to directly affect student learning outcomes. Independent samples *t*-tests and ANOVA were used to answer the research questions.

Results

The first research question was answered using independent samples *t*-tests comparing middle and high school teacher data using the 25-item mathematics content test, 40-item attitudinal test, and 21-item MTEBI with two subscales: PMTE and MTOE. The results of the independent samples *t*-test for the first part of research question one revealed a statistically significant difference between middle school teacher scores and high school teacher scores for the mathematics content pretest (see Table 1). Additionally, there was a large effect size. The results of the independent samples *t*-test for the first part of research question one also revealed a statistically significant difference between middle school teacher scores and high school teacher scores for the mathematics content posttest (see Table 1). Additionally, there was a large effect size. This means high school teachers had higher content test scores than middle school teachers on the pretest and posttest. For attitudes toward mathematics and concepts of self-efficacy there were no statistically significant differences found between middle and high school teachers on both pretest and posttest.

Table 1
Independent Samples t-Test Results on Mathematics Content Test

Assessment	Mean	SD	*t*-value	Effect Size
Mathematics Content Pretest				
Middle School (*N* =26)	68.42	15.600	-3.334**	1.056
High School (*N* = 16)	85.13	16.041		
Mathematics Content Posttest				
Middle School (*N* =26)	79.46	15.402	-3.230**	1.112
High School (*N* = 16)	92.63	6.582		

$N = 42$, $df = 40$, two-tailed
** $p < 0.01$

The second research question was answered using independent samples *t*-tests comparing Mathematic Immersion and Mathematics Teaching Fellows data also using the 25-item mathematics content test, 40-item attitudinal test, and 21-item MTEBI with two subscales: PMTE and MTOE. The results of the independent samples *t*-test for the first part of research question two revealed a statistically significant difference between Mathematics Immersion Teaching Fellows' scores and Mathematics Teaching Fellows' scores for the mathematics content pretest (see Table 2). Additionally, there was a large effect size. The results of the independent samples *t*-test for the first part of research question two also revealed a statistically significant difference between Mathematics Immersion Teaching Fellows' scores and Mathematics Teaching Fellows' scores for the mathematics content posttest (see Table 2). Additionally, there was a large effect size. This means Mathematics Teaching Fellows had higher content test scores than Mathematics Immersion Teaching Fellows on the pretest and posttest. For attitudes toward mathematics and concepts of self-efficacy there were no statistically significant differences found between Mathematics and Mathematics Immersion Teaching Fellows on both pretest and posttest.

Table 2
Independent Samples t-Test Results on Mathematics Content Test

Assessment	Mean	SD	*t*-value	Effect Size
Mathematics Content Pretest				
Mathematics (*N* =30)	68.90	17.008	-4.005**	1.555
Mathematics Immersion (*N* = 12)	89.50	7.868		
Mathematics Content Posttest				
Mathematics (*N* =30)	80.53	14.460	-3.130**	1.202
Mathematics Immersion (*N* = 12)	94.33	7.390		

$N = 42$, $df = 40$, two-tailed
** $p < 0.01$

The third research question was answered using one-way ANOVA comparing different undergraduate college majors also using the 25-item mathematics content test, 40-item attitudinal test, and 21-item MTEBI with two subscales: PMTE and MTOE. Teaching Fellows were grouped according to their undergraduate college major. Three categories were used to group teachers: liberal arts (N = 16), business (N = 11), and mathematics and science (N = 15) majors. The results of the one-way ANOVA for the first part of research question three revealed a statistically significant difference on the mathematics content pretest (see Tables 3 and 4). A post hoc test (Tukey HSD) was performed to determine exactly where the means differed. The post hoc test revealed that mathematics and science majors had significantly higher content knowledge on the pretest than business majors (p = 0.001) and liberal arts majors (p = 0.008). There were no other statistically significant differences. The results of the one-way ANOVA for the first part of research question three also revealed a statistically significant difference on the mathematics content posttest (see Tables 3 and 5). Again, a post hoc test (Tukey HSD) was performed to determine exactly where the means differed. The post hoc test revealed that mathematics and science majors had significantly higher content knowledge on the posttest than business majors (p = 0.005) and liberal arts majors (p = 0.025). There were no other statistically significant differences. It was concluded that mathematics and science majors had statistically significant higher content knowledge scores on both pretest and posttest than non-mathematics and non-science majors. For attitudes toward mathematics and concepts of self-efficacy there were no statistically significant differences found between the undergraduate college majors on both pretest and posttest.

Table 3
Means and Standard Deviations on Content Knowledge

Pretest, Posttest, and CST Test	Mean	Standard Deviation
Content Knowledge Pretest		
Liberal Arts (N = 16)	70.13	16.382
Business (N = 11)	64.45	15.820
Math/Science (N = 15)	87.33	12.804
Total (N = 42)	74.79	17.605
Content Knowledge Posttest		
Liberal Arts (N = 16)	81.19	15.132
Business (N = 11)	76.82	14.034
Math/Science (N = 15)	93.60	7.679
Total (N = 42)	84.48	14.225
CST Content Knowledge		
Liberal Arts (N = 16)	255.81	18.784
Business (N = 11)	249.64	18.943
Math/Science (N = 15)	273.80	15.857
Total (N = 42)	260.62	20.184

Table 4
ANOVA Results on Mathematics Content Pretest for Major

Variation	Sum of Squares	*df*	Mean Square	*F*
Between Groups	3883.261	2	1941.630	8.582**
Within Groups	8823.811	39	226.252	
Total	12707.071	41		

** $p < 0.01$

Table 5

ANOVA Results on Mathematics Content Posttest for Major

Variation	Sum of Squares	*df*	Mean Square	*F*
Between Groups	2066.802	2	1033.401	6.469**
Within Groups	6229.674	39	159.735	
Total	8296.476	41		

** $p < 0.01$

Since significant differences were only found for content knowledge, as measured by the 25-item mathematics content test, it was decided that a focus on content knowledge differences would be appropriate using another content instrument. The first part of each research question was addressed again by using scores on the CST. It was found using an independent samples *t*-test that high school teachers had statistically significant higher content knowledge than middle school teachers as measured by CST scores (see Table 6). Additionally, there was a moderate effect size. Further, it was found using an independent samples *t*-test that Mathematics Teaching Fellows had statistically significant higher content knowledge than Mathematics Immersion Teaching Fellows as measured by CST scores (see Table 6). Additionally, there was a large effect size.

Table 6
Independent Samples t-Test Results on Mathematics Content Specialty Test (CST)

Assessment	Mean	SD	*t*-value	Effect Size
Mathematics CST				
Middle School (N =26)	255.31	20.372	-2.283*	0.741
High School (N = 16)	269.25	17.133		
Mathematics CST				
Mathematics (N =30)	254.33	18.291	-3.636**	1.277
Mathematics Immersion (N = 12)	276.33	16.104		

$N = 42$, $df = 40$, two-tailed
* $p < 0.05$, ** $p < 0.01$

Teaching Fellows were again grouped according to their undergraduate college majors. The results of the one-way ANOVA revealed a statistically significant difference for the CST scores (see Tables 3 and 7). A post hoc test (Tukey HSD) was performed to determine exactly where the means differed. The post hoc test revealed that mathematics and science majors had significantly higher content knowledge, as measured by the CST, than business majors (p = 0.004) and liberal arts majors (p = 0.021). Again, it can be concluded that mathematics and science majors had statistically significant higher content knowledge scores than non-mathematics and non-science majors, as measured by the CST. There were no other statistically significant differences.

Table 7
ANOVA Results on Mathematics Content Specialty Test (CST) for Major

Variation	Sum of Squares	*df*	Mean Square	*F*
Between Groups	4302.522	2	2151.261	6.765**
Within Groups	12401.383	39	317.984	
Total	16703.905	41		

** $p < 0.01$

Discussion and Implications

In a previous study with the same sample it was found that teachers had positive attitudes toward mathematics and high concepts of self-efficacy. Taking the results of the first study with the results found in this present study, a very interesting finding emerged. Teachers had the same high positive attitudes toward mathematics and same high concepts of self-efficacy regardless of content ability. Thus, teachers believed they were just as effective at teaching mathematics, despite not having the high level of content knowledge that some of their colleagues possessed. This is significant since high content knowledge is a necessary condition for quality teaching (Ball et al., 2005).

This study informs teacher education since it found that high school teachers, Mathematics Teaching Fellows, and those who majored in mathematics and science had higher mathematics content knowledge on two measures. Since New York State holds the same high standards for both high school and middle teachers, strategies to better middle school teachers' content knowledge should be investigated and implemented. It is recommended that middle school teachers be given the support they need in mathematics content knowledge by both the schools in which they teach and the schools of education in which they are enrolled.

In order to make well informed decisions about teacher recruitment and development, more research is necessary on the growing alternative certification segment of the teaching population. Unless something is done to better prepare teachers with the rigorous content they need, having teachers who have not majored in mathematics or science related areas teach mathematics could be a disservice to the many urban students who receive instruction from alternatively certified mathematics teachers.

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DETERMING TEACHER QUALITY IN TEACH FOR AMERICA ALTERNATIVE CERTIFICATION TEACHERS

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It is important to understand the relationships between content knowledge and self-efficacy in new teachers. The purpose of this study was to understand the level of self-efficacy and differences between content knowledge and self-efficacy among teachers of different undergraduate majors in the Teach for America program. After taking mathematics and self-efficacy tests, findings revealed that teachers had high levels of self-efficacy. Mathematics related majors had higher mathematical content knowledge than business majors, but similar self-efficacy levels. Liberal arts majors had similar content knowledge and levels of self-efficacy as mathematics related majors.

This research is a follow-up study to a previous study conducted with first year Teach for America (TFA) teachers in New York (Evans, 2009). The previous study found a significant increase in both mathematical content knowledge and positive attitudes toward mathematics over the TFA teachers' first year teaching. Teachers' reflective journals revealed that they generally believed an emphasis on social justice in their coursework was of biggest benefit to them, and that classroom management was the biggest problem faced in their teaching. Additionally, it was found that mathematics related majors had significantly better content knowledge scores on the pre- and posttests and better attitudes toward mathematics on a pretest than did business majors. The purpose of this present study was to understand the level of teacher self-efficacy and differences between content knowledge and self-efficacy among teachers of different undergraduate majors in the TFA program.

Need for the Study

TFA is a non-profit organization formed in 1990 with the intention of sending college graduates to low-income schools to make a difference for underserved students (Kopp, 2003). There have been several prominent studies conducted on TFA teachers in the elementary schools specifically (Darling-Hammond, 1994, 1997; Darling-Hammond, Holtzman, Gatlin, & Heilig, 2005; Laczko-Kerr & Berliner, 2002), but not at the secondary level (Evans, 2009; Xu, Hannaway, & Taylor, 2008). Further, most studies focused primarily on student achievement and teacher retention, admittedly two of the most important variables. However, examining only these variables is not sufficient if the goal is to increase teacher quality.

Darling-Hammond et al. (2005) found that certified teachers consistently produced significantly higher student achievement gains as compared to uncertified teachers, including typically uncertified TFA teachers. Laczko-Kerr and Berliner (2002) found that students of TFA teachers performed more poorly than students of equally inexperienced, but fully certified, teachers. However, students of uncertified TFA teachers performed the same as students of other uncertified teachers (Darling-Hammond et al., 2005; Laczko-Kerr & Berliner, 2002). Certified TFA teachers, after two to three years of teaching and enrolling in a teacher preparation program, performed just as well as other certified teachers in the field. Darling-Hammond et al. cautioned that upon becoming certified many TFA teachers leave teaching. Few studies have addressed mathematical content knowledge with teacher perceptions of self-efficacy (Jones Newton, Leonard, Evans, & Eastburn, in press; Swars, Daane, & Giesen, 2006; Swars, Hart, Smith, Smith, & Tolar, 2007), and no known studies have addressed this issue in alternative certification. Jones Newton et al. (in press) found a relationship between mathematics content knowledge and perceptions of self-efficacy for elementary preservice teachers taking a mathematics methods course. Swars et al. (2007) found an increase in teacher self-efficacy over the course of an elementary mathematics methods class. It is possible that beliefs about self-efficacy may be a greater variable in quality teaching than content knowledge alone (Ernest, 1989).

Theoretical Framework

Ball, Hill, and Bass' (2005) emphasis on the importance of content knowledge for teachers formed the framework of this study. Ball et al. said, "How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing" (Ball et al., 2005, p. 14). Ball et al. suggested that teachers with high content knowledge could help narrow the achievement gap in urban schools. In New York City in particular, and throughout the United States in general, TFA teachers are often placed in high-need urban schools.

Additionally, Bandura's (1986) construct of self-efficacy theory framed this study's focus on self-efficacy in TFA teachers. Bandura found that teacher self-efficacy can be subdivided into a teacher's belief in his or her ability to teach effectively, and his or her belief in affecting student learning outcomes. Teachers who feel that they cannot effectively teach mathematics and affect student learning are more likely to avoid teaching from an inquiry and student-centered approach with real understanding (Swars et al., 2006).

This current study was grounded in this literature (Ball, Hill, & Bass, 2005; Bandura, 1986) since content knowledge and self-efficacy are integral to the teaching and learning process for teachers and their students. Teachers with higher levels of content knowledge and self-efficacy are better able to produce high student achievement than are teachers with lower levels. This study expands upon the literature by examining these constructs among a cohort of new in-service TFA teachers.

Research Questions

4. What level of self-efficacy did TFA teachers possess?
5. Was there a difference in mathematical knowledge between undergraduate majors for TFA teachers?
6. Was there a difference in perceptions of self-efficacy between undergraduate majors for TFA teachers?

Methodology

The sample in this quantitative study consisted of 22 mathematics middle and high school TFA teachers in their second year of teaching and enrollment in a graduate teacher education program with TFA and their partnering university, a large urban university located in New York. For mathematical content knowledge the sample was the entire 22 teachers. However, when self-efficacy was examined the sample was reduced to 19 teachers since two teachers who agreed to participate in the study did not return their self-efficacy instruments, and one teacher left teaching and the TFA program all together in the second year.

Undergraduate majors for teachers consisted of liberal arts (N = 8), business (N = 9), and mathematics related majors (N = 5). This study followed these teachers through their first two years of teaching while completing their graduate teacher education program.

Teachers took the New York State Content Specialty Test (CST), a test required by New York for teacher certification, the summer before they began their program. The range of possible scores on the CST is 100 to 300, and the minimum passing score is 220. Teachers were given a self-efficacy survey in their second year of teaching and graduate education program. The self-efficacy instrument was adapted from the Mathematics Teaching Efficacy Beliefs

Instrument (MTEBI) developed by Enochs, Smith, and Huinker (2000), and measures perceptions of self-efficacy. The MTEBI is a 21-item 5-point Likert scale instrument with choices of strongly agree, agree, uncertain, disagree, and strongly disagree, and is grounded in the theoretical framework of Bandura's self-efficacy theory (1986). Based on the Science Teaching Efficacy Belief Instrument (STEBI-B) developed by Enochs and Riggs (1990), the MTEBI contains two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE) with 13 and 8 items, respectively. Possible scores range from 13 to 65 on the PMTE, and 8 to 40 on the MTOE. The PMTE specifically measures a teacher's self-concept of his or her ability to effectively teach mathematics. The MTOE specifically measures a teacher's belief in his or her ability to directly affect student learning outcomes. Enochs et al. (2000) found the PMTE and MTOE had Cronbach alpha coefficients of 0.88 and 0.77, respectively.

Results

Research question one was answered using independent samples *t*-tests (see Table 1). TFA teachers had statistically significant higher scores on both the PMTE and MTOE than neutral values coded as "2" in the data. Further, the effect sizes for both PMTE and MTOE were very large, and this means that TFA teachers had high levels of self-efficacy. It should be noted, however, that comparing actual self-efficacy scores with neutral responses should be interpreted with caution.

Table 1.

Independent Samples t-Test Results on MTEBI (PMTE and MTOE) Scores

Assessment	Mean	SD	*t*-value	Effect Size
PMTE Actual Scores	3.01	0.320	-13.725**	4.47
Neutral Scores	2.00	0.000		
MTOE Actual Scores	2.85	0.394	-9.381**	3.05
Neutral Scores	2.00	0.000		

$N = 19$, $df = 18$, two-tailed
Equal variances not assumed.
** $p < 0.01$

Research question two was answered using a one-way ANOVA (see Tables 2 and 3). TFA teachers were grouped according to their undergraduate college majors, and three categories were used to group teachers: liberal arts ($N = 8$), business ($N = 9$), and mathematics related (N = 5) majors. For mathematical content knowledge, the one-way ANOVA revealed a statistically significant difference. A post hoc test (Tukey HSD) was performed to determine exactly where the means differed and revealed that mathematics related majors had significantly higher mathematical content knowledge as measured by the CST than did business related majors, $p < 0.05$. There were no other statistically significant differences.

Table 2.

Means and Standard Deviations on Mathematical Knowledge (CST Scores)

CST Scores	Mean	Standard Deviation
Content Proficiency Pre Test		
Liberal Arts (N = 8)	272.88	14.177
Business (N = 9)	255.22	20.891
Mathematics (N = 5)	285.00	20.149
Total (N = 22)	268.41	21.407

Table 3.

ANOVA Results on Mathematical Knowledge (CST Scores) for Major

Variation	Sum of Squares	*df*	Mean Square	*F*
Between Groups	3100.888	2	1550.444	4.516*
Within Groups	6522.431	19	343.286	
Total	9623.318	21		

* $p < 0.05$

Research question three was answered using a one-way ANOVA. No statistically significant differences were found between the various undergraduate college majors and perceptions of self-efficacy as measured by the MTEBI with two subscales: PMTE and MTOE. This means there were no differences between college major and perceptions of self-efficacy.

Discussion and Implications

It was found that TFA teachers had high levels of teaching self-efficacy, which means that teachers had strong beliefs in their ability to teach effectively and affect student learning outcomes. This finding has particularly interesting implications since the literature shows teachers tend to have high levels of student outcome expectancy while they were pre-service teachers. However, outcome expectancy generally declines when the teachers become in-service and the realities of the classroom are encountered (Swars et al., 2007). Teachers in this study had high levels of outcome expectancy despite being in-service teachers. It is possible that TFA teachers are a unique group with higher than usual confidence in their teaching due to the highly selective nature of the TFA program. As previously stated, TFA teachers are generally high achievers coming from very selective universities. This should be further investigated in future research for alternative certification in-service teachers. Comparisons of self-efficacy should be made between TFA teachers and other categories of teachers.

Mathematics related majors had higher mathematical knowledge than did business majors as measured by the CST. This was consistent with the results found in the previous study (Evans, 2009). Similarly, in the previous study there were no differences found between mathematics related majors and liberal arts majors. A possible explanation is that mathematics taught to business majors may be different from mathematics taught to liberal arts and mathematics majors. Mathematics in liberal arts and mathematics programs may be more traditionally academic and aligned with the content taught in middle and high school, whereas business mathematics may be taught from an applications perspective.

There are several implications from these results. First, although mathematics related majors had higher mathematical content knowledge than did business majors, no differences were found in their perceptions of their ability to effectively teach mathematics or their beliefs in their abilities to directly affect student learning outcomes. This is interesting because despite mathematics related majors having higher mathematical ability than business majors, it appears that there is no effect on their perceptions of their ability to teach mathematics effectively and for their students to learn from them. There is a concern that teachers coming from backgrounds other than mathematics related fields do not have enough mathematics content knowledge to effectively teach mathematics. The findings of this study showed that even though a difference was found for content knowledge between the two majors, perceptions of teaching ability were not found to be different. This is significant since self-efficacy is an important variable in quality teaching (Bandura, 1986; Ernest, 1989). Future research should investigate what effect this has on student achievement.

Second, no differences in mathematical ability or perceptions of self-efficacy were found between mathematics related majors and liberal arts majors. The implication is that one does not need to have a mathematics related undergraduate major in order to have sufficient content knowledge and perception of one’s ability to effectively teach mathematics. This indicates that for the TFA teachers who participated in this study it did not matter whether they were mathematics and engineering majors or history, music, political science, psychology, public policy, sociology, and Spanish majors. This could have significant implications for future selection of TFA candidates, and candidates from other alternative certification programs as well. This is an important issue that should be further investigated. Additionally, future research should investigate how student achievement compares between students of teachers from both liberal arts and mathematics backgrounds.

There are several limitations in this study. First, the results of comparing survey results to neutral responses should be interpreted with caution. Levels of self-efficacy should be further compared to preservice and in-service teachers in future studies. Second, only one instrument was used to measure content knowledge. Additional measures of content knowledge should be conducted in future studies, which would eliminate any limitations between test performance and actual content knowledge. Third, the small sample size in this study is a limitation and future studies should replicate this study with larger samples sizes. Finally, future studies should examine the relationships between content knowledge, self-efficacy, and high quality instruction.

Given the need for high quality mathematics teachers, particularly in high-needs urban schools, it is imperative that students in these schools are getting the quality education they deserve. To make sure this is happening we must continuously examine teacher quality in teacher preparation in traditional programs and especially alternative pathways programs such as TFA, to ensure that all children have the highest quality teachers.

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CHARACTERIZING AN ORIENTATON TOWARD LEARNING AND TEACHING MATHEMATICS THAT CONSTRAINS REFLECTION[1]

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The goal of this study was to investigate in-service mathematics secondary school teachers' ways of thinking that supported or constrained their capacity to reflect on their practice as they engaged in activities designed to promote powerful mathematical knowledge for teaching as proposed by Silverman and Thompson (2008). I propose an orientation toward learning and teaching of mathematics (an empirical orientation) as a potential way of thinking that accounts for teachers' ways of operating and helps to explain the teachers' reticence to reflect.

Stigler and Hiebert (1999) drawing on the conclusions of the Third International Mathematics and Science Study (TIMSS), highlighted the necessity for reform of mathematics teaching in the United States. In the years since *The Teaching Gap* there has been one generally agreed upon theme—that students are not developing a satisfactory level of mathematical proficiency (e.g., Gonzales, et al., 2004).

Although elementary and secondary students' mathematics performance has shown some improvement over the past decade, this improvement has not been in all grades assessed and is not equal for all groups of students (e.g., Hall & Kennedy, 2006). Several documents have indicated the important role that teachers play, not only in what students learn, but also in any mathematics reform effort (e.g., National Mathematics Advisory Panel, 2008). Therefore, in order to most propitiously influence student learning, it is of paramount importance to identify characteristics of effective teaching and determine how best to develop these characteristics in the minds of teachers.

Beginning with Shulman's (1986) seminal work regarding pedagogical content knowledge (PCK), several researchers have focused on the form, nature, organization, and content of teachers' mathematical knowledge (e.g., Lampert, 1991). One recent research trend is to acknowledge that most of what Shulman (1986) had in mind is really mathematical knowledge that has special characteristics that support the teaching of mathematics. As such, the phrase mathematical knowledge for teaching (MKT), initiated with the work of Thompson and Thompson (1996), is now often used in place of pedagogical content knowledge (e.g., Ball, Thames, & Phelps, 2008). In Silverman and Thompson's (2008) view, the development of powerful mathematical knowledge for teaching involves developing significant personal understandings of a particular mathematical topic and transforming these personal understandings to understandings and ways of thinking that are pedagogically powerful. According to Silverman and Thompson (2008), both the personally and pedagogically powerful understandings develop via a process that Piaget (2001) called reflective abstraction.

The current study explored in-service secondary school mathematics teachers' cognitions as they engaged in activities designed to promote powerful mathematical knowledge for teaching as proposed by Silverman and Thompson (2008).

Methodology

Data were generated during a graduate level mathematics education course on conceptual approaches to teaching major ideas in secondary mathematics. The course and study were part of the NSF-sponsored Teachers Promoting Change Collaboratively (TPC^2) Project, conducted by Professor Patrick Thompson and his research team at Arizona State University. The larger project goals were to help teachers move from a very teacher-centered orientation to a very student-centered orientation, and to develop models of teachers who are invested in students' understanding and for influencing teaching practices. Project participants consisted of 16 high school mathematics teachers from a large district in the southwestern United States. Data consisted of videotape of all 14-class sessions, work artifacts created by course participants, and field notes of relevant participant observations made by project research assistants.

The course was designed to both promote and investigate the development of mathematical knowledge for teaching in the Silverman and Thompson (2008) framework. As such, course activities were designed explicitly to create contexts for which teachers would reflect on their own activity in a way that might translate into them doing things differently in their own classrooms.

Data analysis happened at three levels: (1) Review entire video collection with the intent of identifying occasions where instruction was designed for teachers to transform their practices via reflective abstraction (designated "reflective episodes"). Salient selection criterion was that the episode be designed such that teachers were provoked to reflect on their ways of operating; (2a) Provide descriptions of and rationale for each reflective episode (this provided context for my analysis and models of the course instructor's intended instructional outcomes); (2b) Construct preliminary models of teachers' ways of thinking as they engaged with instruction (reflective episodes and larger data corpus); and, (3) Modify or adjust models of teachers' ways of operating developed in level 2 as the teachers engaged with instruction (reflective episodes).

Empirical Orientation Toward Learning and Teaching Mathematics

In my analysis, as I explored how teachers engaged with instruction designed for them to re-think their teaching, re-conceptualize their students, or re-conceive their mathematics, I encountered one reoccurring theme—participating teachers consistently demonstrated a tendency not to reflect on their practice. I found very few instances that suggested any kind of reflection on the part of teachers though they had many opportunities (from my perspective) where it not only would have been natural to do so, but instruction was designed so that they would. I saw few indications of teachers questioning their own or their colleagues' meanings, questioning their assumptions, questioning the coherence of their meanings, and questioning whether their meanings were aligned or even compatible with those of their students or colleagues.

I propose an orientation toward learning and teaching mathematics (an empirical orientation) that accounts for both teachers' personal and pedagogical ways of operating. I will first attempt to operationalize an empirical orientation and then demonstrate how the construct accounts for teachers' actions as they engaged with instruction designed for them to reflect.

An empirical orientation is characterized by a platonic ontological view of mathematics, and is egocentric and empirical in nature. By empirical, I mean an orientation that is not only guided by perception, but focuses on the visible. By a focus on the visible, I mean an orientation toward objects that can be perceived in some manner, such as, but not limited to visually and orally. One way that such an orientation is expressed is through instruction that focuses students' attention on the visible—a focus on action sequences involving procedures, skills, and facts to arrive at an outcome; rather than a focus on reasoning.

Boyd (1992) characterizes one mathematics teacher's subject matter knowledge in terms of objects and actions the teacher envisioned being applied to those objects. This characterization is compatible with the mathematical conceptions of a teacher with an empirical orientation.

An empirical orientation places an emphasis on providing students with opportunities to understand objects by engaging in tasks that involve performing actions on those objects, although the focus is not on actions (i.e., reasoning). Rather, the focus is on performing a sequence of steps to arrive at (and perhaps justify why certain steps are allowed) an outcome. The focus is on the product of reasoning, where the reasoning itself stays hidden.

An empirical orientation places an emphasis on getting students to attain "the" understanding to concepts (i.e., how the teacher has conceptualized the mathematics). Such an emphasis makes the role of interpretation (the idiosyncratic understandings of others) hidden to the teacher. In addition, a focus on the visible emphasizes knowledge of the empirical, that is, knowledge that is figural (Piaget, 1976), which is static. Being static, the actions producing figural knowledge are not easily visible to the person having it. Finally, a focus on objects (i.e., the things that are perceived or imagined), rather than actions (i.e., reasoning) limits abstractions to those based on the objects themselves.

Teachers' Attempts To Reflect

A reflective episode from Class #12 was designed to have teachers act, over the class period, with two hats. The first was with the hat of a student of a lesson in which teachers attempted to make sense of a dynamic situation involving an invariant relationship (i.e., always speeding up) between two covarying quantities (i.e., distance traveled and elapsed time); the second was with the hat of an instructional designer creating the lesson that they just experienced. This lesson built on an earlier reflective episode (spanning Classes #3 and #4) designed to move teacher's from a conception of speed as one identified by the relationship speed = Δdistance/Δtime, to a scheme of meanings, including: (1) speed as a quantification of motion, (2) completed motion as involving two completed quantities—distance traveled and amount of time required to travel that distance; and, (3) constant speed as a direct proportional relationship between distance traveled and the amount of time required to travel that distance.

The course instructor (Pat Thompson) initiated the lesson (Class #12) by standing on a chair, dropping a screwdriver, and asking, "What did you see?" Pat managed the conversation in a manner that motivated a discussion of the object's speed by focusing teachers' observations on the screwdriver "as" it fell; specifically, whether the screwdriver had fallen at a constant speed.

Pat next moved teachers to make explicit their images of what it would mean for the screwdriver to have fallen at a constant speed. Tami (a participating teacher) responded, "The amount that it falls would be the same for every increment of time, so if you split a second into ten parts, the amount that it fell would be exactly the same in each increment." Tami's response suggests that she may have been thinking of constant speed as an object whose component parts (distance traveled and elapsed time) stay in constant proportion.

Pat moved the conversation toward determining a way that would allow teachers to answer the question of whether the screwdriver fell at a constant speed, experimentally, rather than relying on what they "knew" to be true (accelerating due to gravity) or what they visually perceived. Two teachers (Rachel and Tami) each proposed that they calculate the time that the screwdriver fell for two distinct distances, divide distance traveled by elapsed time for each pair of data (to obtain "speeds"), and then compare these two calculated values. These proposals suggest that each teacher was thinking about speed as the object obtained after dividing distance traveled by elapsed time. This conception differs from how Tami earlier described constant speed, suggesting that Tami might have been reflecting on her conception of constant speed.

Pat restated Tami's original explanation for what it would mean were the screwdriver to fall at a constant speed, and asked again how they could test this definition. Rachel asserted that, "[They] could…[2]start…with an increment here [places hand out], time that. Double that [moves hand up] time it again to see if…then take that up another, whatever that increment is." Rachel's comment suggests that she may have been thinking of constant speed as a multiplicative relationship between distance traveled and the time taken to travel that distance. Such a conception differs from how Rachel earlier expressed her understanding, suggesting that she might have been reflecting.

Pat stated that they were going to take several measurements from two heights (260 cm and 130 cm), look at the distributions of time, and make estimates of the speeds. After recording several trials, teachers determined that the screwdriver had taken 0.7 second to fall 260 cm, and 0.5 second to fall the first 130 cm. Pat asserted that based on these results, the screwdriver's speed was not constant, and that the screwdriver was going faster in the second half of its fall.

Pat requested that teachers develop (in groups) a meaning for "it was always speeding up." All groups either focused their attention on comparing distances (for equal increments of time elapsed), such as "For equal increments of time, the distance traveled is greater for each consecutive increment," or comparing times (for equal increments of distances traveled). Such conceptions focus on the comparison and on obtaining a result of the comparison—a focus on the objects (the two "speeds"), the action ("comparing") on those objects, and the result of the comparison (the product of reasoning). In addition, none of the groups developed a meaning for "always speeding up" that cohered with their meaning for speed and that allowed for comparisons of any increment of distance traveled and any increment of time elapsed.

Pat next requested that teachers construct a graph of the screwdriver's distance from the ground in relation to the elapsed time since it was dropped so that their graph satisfied four constraints: (1) the screwdriver's initial height was 260 cm, (2) it took 0.7 second to drop that height, (3) at 0.5 second the screwdriver was 130 cm off of the ground, and (4) the screwdriver was always speeding up.

After observing for several minutes, Pat asserted that the majority of teachers were not looking at any increments of time; they were thinking about a start time, an end time, that the graph must be curved, and simply drawing a parabola that passed through the points (0, 260), (0.5, 130), and (0.7, 0). Pat asserted that teachers were not making their graph reflect the information in their statement for "it was always speeding up." This suggests that teachers had assimilated the task as one involving the object "quadratic function," and an associated action performed on that object, where the action involved constructing a graph of a quadratic function representing the contextual situation. In addition, this suggests that the teachers were focused on the outcome of the activity (obtaining the graph), and were not reasoning about the situation—they were not coordinating their actions (i.e., meanings) to construct the graph.

Throughout the lesson, teachers demonstrated the fragile nature of the meanings (related to the concept of speed) that they had developed during earlier class sessions. Teachers appeared constrained in their capacity to recall the meanings and understandings that they had worked to transform. In addition, teachers consistently focused on the products of their reasoning, rather than their reasoning—their reasoning was hidden in their products. Finally, teachers were focused on their perceptions, their experiences in the lesson, rather than their meanings and the need to have ideas and meanings develop and cohere—they were disinclined to take their meanings, their reasoning, as objects of thought.

As a large group, teachers next attempted to re-construct the logic of the lesson, a move which was designed to provoke reflection on the instructional actions that teachers might take to support student development of the intended meanings and ideas, and the reasons why those actions might work—including the tasks and classroom discourse that the teacher would employ. Specifically, teachers were asked to answer the question, "How did we get from standing on a chair dropping a screwdriver, to discussing these graphs." Although teachers were able, for the most part, to re-construct the lesson, they did so by simply re-constructing the sequence of events. What teachers were constrained to re-construct was the lesson's logic. Throughout the lesson re-construction, there was little discussion of the actual meanings that would either guide their instructional actions or that would be desired learning outcomes for students. Rather than re-constructing the logic of the lesson, the meanings that the lesson intended to build, the ideas that the lesson intended to develop, and teacher's orchestration of the lesson to promote these intended understandings and ways of thinking in their students, teachers focused their attention on re-constructing the sequence of successive outcomes (the step by step products of teachers' reasoning).

Discussion

As the reflective episode illustrates, an empirical orientation toward learning and teaching mathematics accounts for teachers' focused attention on their own perceptions of the lesson that they had engaged with as students, on objects and actions that they perceived as being applied to those objects, and on the products of their reasoning. Such a focus constrained teachers' capacity to take their meanings as objects of thought and to take the point of view of others. Furthermore, such an orientation focused teachers' attention on the visible (e.g., comparing speeds, the graph); thus, making the idea of creating and implementing a lesson possessing a logic, one that systematically develops and builds meanings and ideas, hidden to teachers.

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[2] In the excerpt sections, the symbol "…" signifies that either the speaker paused during the utterance, or that the speaker did not complete the utterance due to an interruption or because they simply stopped speaking.

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